8.5 Partial fractions

1. Divide if improper.
2. Factor denominator.
3. Decompose into partial fractions.

Heaviside's Method:
1. Cover one factor
2. Evaluate with the covered factor's root.
3. The value is the numerator for the covered factor.

ex. $\dfrac{x+5}{(x+2)(x-1)} = \dfrac{-1}{x+2} + \dfrac{2}{x-1}$

$-2 \qquad\qquad 1$

$\dfrac{-2+5}{-2-1} = \dfrac{3}{-3} = -1 \qquad \dfrac{1+5}{1+2} = \dfrac{6}{3} = 2$

$\dfrac{A(x-1)}{(x+2)(x-1)} + \dfrac{B(x+2)}{(x-1)(x+2)}$

$\underline{Ax} - \underline{A} + \underline{Bx} + \underline{2B} = \underline{x} + \underline{5}$

$x: A + B = 1$

Cnst $-A + 2B = 5$

ex. Decompose into partial fractions: $\frac{9x-4}{3x^2+x-10}$

$\frac{3}{3x-5} + \frac{2}{x+2}$

$(3x-5)(x+2)$

$\frac{5}{3}$ $\quad$ -2

$\frac{9(\frac{5}{3})-4}{\frac{5}{3}+\frac{2}{1}} = \frac{15-4}{\frac{11}{3}} = \frac{11}{\frac{11}{3}} = 11 \cdot \frac{3}{11}$

$= 3$

$\frac{9(-2)-4}{3(-2)-5} = \frac{-18-4}{-6-5} = \frac{-22}{-11} = 2$

ex. $\int \frac{12x+13}{8x^2+14x+3}\,dx$

$= \int \frac{4\,dx}{4x+1} + \frac{1}{2}\int \frac{2 \cdot 1\,dx}{2x+3}$

M $8 \cdot 3 = 24$

A $14 = \underline{2} + \underline{12}$

R $\underline{8x^2+2x} + \underline{12x+3}$

F $2x(4x+1) + 3(4x+1)$

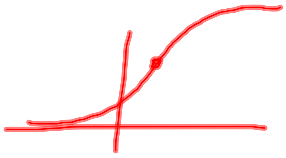
F $(4x+1)(2x+3)$

$= \ln|4x+1| + \frac{1}{2}\ln|2x+3| + C$

$\frac{12\left(-\frac{3}{2}\right)+13}{4\left(-\frac{3}{2}\right)+1} = \frac{-5}{-5} = +1$

$\frac{12\left(-\frac{1}{4}\right)+13}{2\left(-\frac{1}{4}\right)+3} = \frac{10}{\frac{5}{2}} = 4$

ex. Solve $\dfrac{dP}{dt} = 0.08P\left(1-\dfrac{P}{1000}\right)$ if $P_0=100$



http://www.math.neu.edu/~gilmore/U343su05files/logistic.pdf

$$\frac{1}{1000}\cdot\frac{dP}{P\left(1-\frac{P}{1000}\right)} = .08\,dt\cdot\frac{1}{1000}$$

$$\frac{1\,dP}{P(1000-P)} = .00008\,dt$$

$$\frac{\frac{1}{1000}dP}{P} + \frac{\frac{1}{1000}dP}{1000-P} = .00008\,dt$$

$$\int\frac{1\,dP}{P} \overset{-}{+} \int\frac{-1\,dP}{1000-P} = \int .08\,dt$$

$$\ln P - \ln(1000-P) = .08t + C_1$$

$$-\ln P + \ln(1000-P) = -.08t + C_2$$

$$\ln\frac{1000-P}{P} = -.08t + C_2$$

$$\frac{1000-P}{P} = e^{-.08t+C_2}$$

$$\frac{1000-P}{P} = C\cdot e^{-.08t}$$

$t \quad P$

$(0, 100)$

$$\frac{1000-100}{100} = Ce^{0}$$

$$9 = C$$

$$\frac{1000}{P} - 1 = 9e^{-.08t}$$

$$\frac{1000}{P} = \frac{1+9e^{-.08t}}{1}$$

$$\frac{P}{1000} = \frac{1}{1+9e^{-.08t}}$$

$$\frac{P}{1} = \frac{1000}{1+9e^{-.08t}}$$