8.7 Indeterminate forms and L'Hôpital's Rule

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L'Hôpital's Rule:

f and g are diff'ble on (a,b)
except possibly at c

g'(x)≠0 for all x in (a,b)
except possibly at c

$\lim_{x \to c} \frac{f(x)}{g(x)}$ is an indeterminate form
(0/0, ∞/∞, etc.)

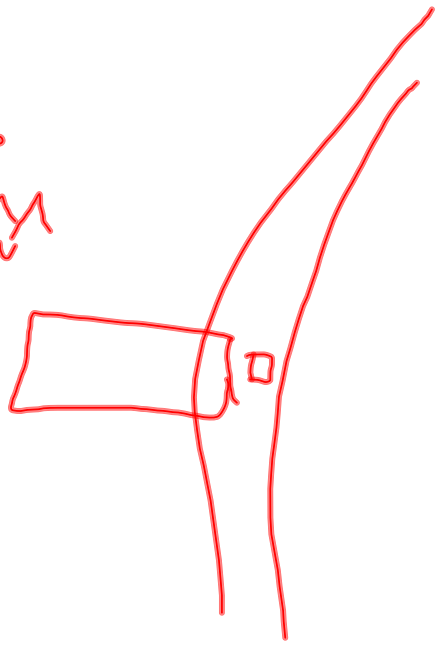
$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$$

ex. $\lim_{x\to0} \frac{\sin 3x}{x} = \frac{0}{0}$

$= \lim_{x\to0} \frac{3\cos 3x}{1} = \frac{3\cdot 1}{1} = 3$

ex $\lim_{x\to0} \frac{e^{2x}-1}{x} = \frac{1-1}{0} = \frac{0}{0}$

$\lim_{x\to0} \frac{2e^{2x}}{1} = \frac{2\cdot 1}{1} = 2$



ex $\lim_{x\to\infty} \frac{\ln x}{x} = \frac{\infty}{\infty}$

$\lim_{x\to\infty} \frac{\frac{1}{x}}{1} = \lim_{x\to\infty} \frac{1}{x} = 0$

ex. $\lim_{x\to -\infty} \frac{x^2}{e^{-x}} = \frac{\infty}{\infty}$

$\lim_{x\to -\infty} \frac{2x}{-e^{-x}} = \frac{-\infty}{-\infty}$

$\lim_{x\to -\infty} \frac{2}{e^{-x}} = 0$

ex. $\lim_{x\to\infty} \frac{e^{-x}\sqrt{x}}{1} = \lim_{x\to\infty} \frac{\sqrt{x}}{e^{x}} = \frac{\infty}{\infty}$

$= \lim_{x\to\infty} \frac{\frac{1}{2}x^{-\frac{1}{2}}}{e^{x}}$

$= \lim_{x\to\infty} \frac{1}{2\sqrt{x}\,e^{x}} = 0$

ex. $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = 1^\infty$

$y = \left(1+\frac{1}{x}\right)^x$

$\ln y = x \ln\left(1+\frac{1}{x}\right)$

$\lim_{x\to\infty} x^1 \ln\left(1+\frac{1}{x}\right)$

$\infty \cdot 0$

$\lim_{x\to\infty} \frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}} = \frac{0}{0}$

$x^{-1} \to \frac{1}{x}$

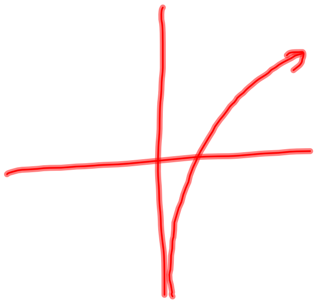
$\lim_{x\to\infty} \frac{\frac{1}{\left(1+\frac{1}{x}\right)} \cdot \cancel{(-x^{-2})}}{\cancel{(-x^{-2})}}$

$\lim_{x\to\infty} \frac{1}{\left(1+\frac{1}{x}\right)} = 1$

$\ln y = 1$

$y = e^1 = e$

ex. $\lim\limits_{x\to 0^+} (\sin x)^x = 0^0$



$\ln y = x \ln \sin x$

$\lim\limits_{x\to 0^+} x \ln \sin x$

$0(-\infty)$

$\lim\limits_{x\to 0^+} \dfrac{\ln \sin x}{\frac{1}{x}} = \dfrac{-\infty}{\infty}$

$= \lim\limits_{x\to 0^+} \dfrac{\frac{1}{\sin x}\cdot \cos x}{-\frac{1}{x^2}}$

$\lim\limits_{x\to 0^+} \dfrac{\cos x \cdot (-x^2)}{\sin x}$ $\quad \dfrac{1\cdot 0}{0}$

$\lim\limits_{x\to 0^+} -x \dfrac{\cos x}{1} \cdot \dfrac{x}{\sin x}$

$-0 \cdot 1 \cdot 1 = 0$

$\lim\limits_{x\to 0} \dfrac{\sin x}{x} = 1$

$\ln y = 0$

$y = e^0 = 1$