8.8 Improper Integrals

Defn.

1. If f is continuous on [a,∞) then

$$\int_a^{\infty} f(x)dx = \lim_{b\to\infty} \int_a^b f(x)dx$$

2. If f is continuous on (-∞,b] then

$$\int_{-\infty}^{b} f(x)dx = \lim_{a\to-\infty} \int_a^b f(x)dx$$

3. If f is continuous on (-∞,∞) then

$$\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{c} f(x)dx + \int_c^{\infty} f(x)dx$$

If the limit exists then the integral converges, otherwise, it diverges (tends toward infinity).

ex. $\int_0^{\infty} e^{-x} dx$

ex. $\int_0^e \ln x \, dx$

ex. $\int_{-\infty}^{\infty} \frac{e^x dx}{1+e^{2x}}$

ex. $\int_1^{\infty} (1-x)e^{-x}\,dx$

Thm. $\int_1^\infty \frac{1}{x^p}dx = \begin{cases} \frac{1}{p-1}, \text{ if } p>1 \\ \text{diverges otherwise} \end{cases}$

ex. Find the volume of the solid formed by revolving the function y=1/x around the x axis for x≥1.

ex. $\int_4^5 \frac{dx}{(x-4)^3}$

ex. $\int_5^\infty \frac{dx}{(x-4)^3}$