9.1 Sequences

Defn. A **sequence** is a function whose domain is the set of natural numbers (the positive integers).

Sequence=$\{a_n\}$

ex. List the first 4 terms of $\left\{a_n = \frac{2^n}{n!}\right\}$

ex. List the first 4 terms of $\left\{a_n = \frac{n^2}{2^n - 1}\right\}$

Limits of sequences: $\lim_{n\to\infty} a_n = L$

If L is a real number, then $\{a_n\}$ converges; otherwise, it diverges.

ex. determine the convergence of each:

$$\left\{a_n = \frac{2^n}{n!}\right\}$$

$$\left\{a_n = \frac{n^2}{2^n - 1}\right\}$$

ex. Write an expression for each sequence:

1, 4, 7, 10, ...

$$\frac{-2}{1}, \frac{8}{2}, \frac{-26}{6}, \frac{80}{24}, \frac{-242}{120}, \ldots$$

ex. Determine monotonicity of $\{\frac{2n}{1+n}\}$

Defn. A sequence $\{a_n\}$ is...

...bounded above if there's an $M > a_n$ for all n.

...bounded below if there's an $m < a_n$ for all n.

...bounded if both of these apply.

Thm. A sequence that is monotonic and bounded must converge.