9.1 Sequences

Defn. A **sequence** is a function whose domain is the set of natural numbers (the positive integers).

Sequence= $\{a_n\}$

ex. List the first 4 terms of $\left\{a_n = \dfrac{2^n}{n!}\right\}$

$\frac{2}{1}, \frac{4}{2}, \frac{8}{6}, \frac{16}{24}$

$2, 2, \frac{4}{3}, \frac{2}{3}$

ex. List the first 4 terms of $\left\{a_n = \dfrac{n^2}{2^n - 1}\right\}$

$n: \quad 1 \quad 2 \quad 3 \quad 4$

$a_n: \frac{1}{1}, \frac{4}{3}, \frac{9}{7}, \frac{16}{15}$

<u>Limits of sequences:</u> $\lim_{n\to\infty} a_n = L$

If L is a real number, then $\{a_n\}$ converges; otherwise, it diverges.

ex. determine the convergence of each:

$$\left\{a_n = \frac{2^n}{n!}\right\} \longrightarrow 0$$

$\frac{2}{1}, 2, \frac{4}{3}, \frac{2}{3}, \frac{32}{120}, \frac{64}{720}$

$\frac{4}{15}$ $\frac{4}{45}$

$$\left\{a_n = \frac{n^2}{2^n - 1}\right\} \longrightarrow 0$$

$1, \frac{4}{3}, \frac{9}{7}, \frac{16}{15}, \frac{25}{31}, \frac{36}{63}$

fastest n^n

Factorial

Exponential

Power /polynomial

Logarithmic

slowest Constant

"n n f e p l constant"

ex. Write an expression for each sequence:

1 2 3

1, 4, 7, 10, ...

3 3 3

$1+3(n-1)$

3 9 27 81 243

$$\frac{-2}{1},\frac{8}{2},\frac{-26}{6},\frac{80}{24},\frac{-242}{120},\ldots$$

1 1·2 1·2·3 1·2·3·4 1·2·3·4·5

$$\frac{(-1)^n(3^n-1)}{n!}$$

$$\boxed{\begin{array}{l}\text{Defn} \\ 0! = 1\end{array}}$$

ex. Determine monotonicity of $\left\{\frac{2n}{1+n}\right\}$

(strictly incr. or strictly decr.)

via derivatives

$f(x) = \frac{2x}{1+x}$

$f'(x) = \frac{2(1+x) - 2x(1)}{(1+x)^2}$

$= \frac{2}{(1+x)^2}$

for $x \geq 1$, $f'(x) > 0$

so the sequence is monotonic increasing.

compare general terms

Assume mono. incr. / decr

$a_n \lesseqgtr a_{n+1}$

$\frac{2n}{1+n} \lesseqgtr \frac{2(n+1)}{1+(n+1)}$

$2n(1+n+1) \lesseqgtr 2(n+1)(n+1)$

$2n(n+2) \lesseqgtr 2(n^2+2n+1)$

$2n^2+4n \lesseqgtr 2n^2+4n+2$

$0 \lesseqgtr 2$

Defn. A sequence $\{a_n\}$ is...

...bounded above if there's an $M>a_n$ for all n.

...bounded below if there's an $m<a_n$ for all n.

...bounded if both of these apply.

Thm. A sequence that is monotonic and bounded must converge.