## 9.2 Series and convergence

Defn. The sum of a sequence is a series.

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \ldots$$

Defn. The nth partial sum is the sum of the first n terms of an infinite series.

Defn. If the sequence of partial sums $\{S_n\}$ converges to a real number S, then the series converges and S is called the limit of the series.

ex. Determine whether each series converges.

$$\sum_{n=1}^{\infty} \frac{1}{2^n}$$

$$\sum_{n=1}^{\infty} 1$$

ex. Determine whether $\sum_{n=1}^{\infty} \frac{2}{4n^2-1}$ converges.

Defn. A Geometric series with common ratio r is of the form $\sum_{n=0}^{\infty} ar^n = a+ar+ar^2+ar^3+\ldots$

Thm. A geometric series with $|r| > 1$ will diverge whereas if $0<|r|<1$ will converge to $\frac{a}{1-r}$.

ex. Determine the convergence:

$\sum_{n=0}^{\infty} \frac{3}{2^n}$

$\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n$

Thm. $\sum a_n$ converges $\longrightarrow$ $\{a_n\}$ converges to 0.

Thm. The nth Term Test (Contrapositive of the above)
$\{a_n\}$ does not converge to 0 $\longrightarrow$ $\sum a_n$ will not converge.

$\{a_n\}$ goes to 0? Y $\rightarrow$ $\sum a_n$ may or may not converge.
N $\rightarrow$ $\sum a_n$ doesn't converge.

ex. What, if anything, does the nth term test tell us about these?

$$\sum_{n=0}^{\infty} 2^n$$

$$\sum_{n=1}^{\infty} \frac{2^n}{n+1}$$

ex. What, if anything, does the nth term test tell us about these?

$$\sum_{n=1}^{\infty} \frac{n!}{2n!+1}$$

$$\sum_{n=1}^{\infty} \frac{1}{n}$$