9.2 Series and convergence

Defn. The sum of a sequence is a series.

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \ldots$$

Defn. The nth partial sum is the sum of the first n terms of an infinite series.

Defn. If the sequence of partial sums $\{S_n\}$ converges to a real number S, then the series converges and S is called the limit of the series.

ex. Determine whether each series converges.

$$\sum_{n=1}^{\infty} \frac{1}{2^n} = 1$$

n	1	2	3	4	5
a_n	$\frac{1}{2}$,	$\frac{1}{4}$,	$\frac{1}{8}$,	$\frac{1}{16}$,	$\frac{1}{32}$
S_n	$\frac{1}{2}$,	$\frac{3}{4}$,	$\frac{7}{8}$,	$\frac{15}{16}$,	$\frac{31}{32}$ $\longrightarrow 1$

$$\sum_{n=1}^{\infty} 1$$

this diverges

a_n	1,	1,	1,	1,	1
S_n	1,	2,	3,	4,	5 $\longrightarrow \infty$

ex. Determine whether $\sum_{n=1}^{\infty} \frac{2}{4n^2-1}$ converges. yes!

n	1	2	3	4	5
a_n	$\frac{2}{3}$,	$\frac{2}{15}$,	$\frac{2}{35}$,	$\frac{2}{63}$,	$\frac{2}{99}$
S_n	$\frac{2}{3}$,	$\frac{12}{15}$,	$\frac{30}{35}$	$\frac{56}{63}$	
		$\frac{4}{5}$,	$\frac{6}{7}$,	$\frac{8}{9}$,	$\frac{10}{11} \to 1$

Defn. A Geometric series with common ratio r is of the form $\sum_{n=0}^{\infty} ar^n = a + ar + ar^2 + ar^3 + \ldots$

Thm. A geometric series with $|r| > 1$ will diverge whereas if $0 < |r| < 1$ will converge to $\frac{a}{1-r}$.

ex. Determine the convergence:

$\sum_{n=0}^{\infty} \frac{3}{2^n}$

$a = 3$

$r = \left|\frac{1}{2}\right| < 1$ this converges to $\frac{3}{1-\frac{1}{2}} = 6$

$3 \xrightarrow{\cdot\frac{1}{2}} \frac{3}{2} \xrightarrow{\cdot\frac{1}{2}} \frac{3}{4}$

$3 + \frac{3}{2} + \frac{3}{4}$

$\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n$

$a = 1$

$r = \left|\frac{3}{2}\right| > 1$

this series diverges

Thm. $\sum a_n$ converges $\longrightarrow$ $\{a_n\}$ converges to 0.

Thm. <u>The nth Term Test</u> (Contrapositive of the above)

$\{a_n\}$ does not converge to 0 $\longrightarrow$ $\sum a_n$ will not converge.

$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots$

$\{a_n\}$ goes to 0? $\xrightarrow{Y}$ $\sum a_n$ may or may not converge.

$\{a_n\}$ goes to 0? $\xrightarrow{N}$ $\sum a_n$ doesn't converge.

$\frac{2}{1} + \frac{3}{2} + \frac{4}{3} + \frac{5}{4} + \frac{6}{5} + \cdots$

ex. What, if anything, does the nth term test tell us about these?

$$\sum_{n=0}^{\infty} 2^n$$

$$\sum_{n=1}^{\infty} \frac{2^n}{n+1}$$

faster

n^n

Factorial

Exponential

Poly

Log

Const

slower

$\lim_{n\to\infty} 2^n = \infty$

& not to 0

so $\sum_{n=0}^{\infty} 2^n$ diverges

$\lim_{n\to\infty} \frac{2^n}{n+1} \neq 0$

so this series won't converge

ex. What, if anything, does the nth term test tell us about these?

$$\sum_{n=1}^{\infty} \frac{n!}{2n!+1}$$

$\frac{1}{3}, \frac{2}{5}, \frac{6}{13}, \frac{24}{49} \to \frac{1}{2} \neq 0$

this series diverges

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

$\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5} \longrightarrow 0$

the n^{th} term test doesn't tell us any thing