9.3 Integral Test

Thm: The integral test

f is positive, continuous, & decreasing for $x \geq 1$ & $a_n = f(n)$

$\sum_{n=1}^{\infty} a_n$ & $\int_1^{\infty} f(x)dx$

either
both converge or
both diverge

Thm: Convergence of p-series

The p-series $\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \ldots$

will converge if $p>1$ and diverge if $0<p\leq 1$.

If $p=1$, we call this the harmonic series.

ex. Determine convergence:

$$\sum_{n=1}^{\infty} \frac{n^2}{n^3+1}$$

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$$\sum_{n=1}^{\infty} \frac{1}{n^2+1}$$

ex. Determine convergence:

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n}}$$

ex. Does $1+\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}+\frac{1}{4\sqrt{4}}+\ldots$ converge?