9.3 Integral Test

Thm: The integral test

f is positive, continuous, & decreasing for $x \geq 1$ & $a_n = f(n)$ $\longrightarrow$ $\sum_{n=1}^{\infty} a_n$ & $\int_1^{\infty} f(x)\,dx$

either both converge or both diverge

Thm: <u>Convergence of p-series</u>

The p-series $\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \ldots$

will converge if $p>1$ and diverge if $0<p\leq 1$.

If $p=1$, we call this the harmonic series.

ex. Determine convergence:

$$\sum_{n=1}^{\infty} \frac{n^2}{n^3+1}$$

$\approx \frac{1}{n}$

$u = x^3 + 1$

$du = 3x^2\,dx$

$$\frac{1}{3}\int_1^{\infty} \frac{3x^2}{x^3+1}\,dx$$

$$\frac{1}{3}\int_1^{\infty} \frac{du}{u}$$

$$= \frac{1}{3} \lim_{b\to\infty} \int_1^{b} \frac{du}{u}$$

$$= \frac{1}{3} \lim_{b\to\infty} \ln u \Big|_1^b$$

$$= \frac{1}{3} \lim_{b\to\infty} \ln x^3+1 \Big|_1^b$$

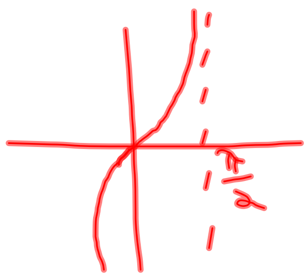
$$= \frac{1}{3}\left(\underbrace{\lim_{b\to\infty} \ln b^3+1}_{\to\,\infty} - \ln 2 \right)$$

this integral diverges and so does the series.

ex. Determine convergence:

$$\sum_{n=1}^{\infty} \frac{1}{n^2+1}$$

$$\int_1^{\infty} \frac{1}{x^2+1}\,dx$$



$$\lim_{b\to\infty} \arctan x \Big|_1^b$$

$$\lim_{b\to\infty} \arctan b - \arctan 1$$

$$= \frac{\pi}{2} - \frac{\pi}{4}$$

the integral & the series both converge

ex. Determine convergence:

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n}}$$

$$\frac{1}{n^{\frac{1}{5}}}$$

pseries with $p = \frac{1}{5}$

so this series diverges

ex. Does $1+\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}+\frac{1}{4\sqrt{4}}+\ldots$ converge?

$$\frac{1}{1^{1\frac{1}{2}}}+\frac{1}{2^{1\frac{1}{2}}}+\frac{1}{3^{3/2}}+\frac{1}{4^{3/2}}$$

p series with $p=\frac{3}{2}$

so it converges.