

9.4 Direct Comparison Test

Thm. Direct Comparison Test

Let $0 < a_n < b_n$ for all n beyond some value.

if the series of
this term diverges,

if the series of
this term converges,

then so does the other series.

ex. Determine the convergence of $\sum_{n=1}^{\infty} \frac{1}{5+3^n}$

Compare to

$$\sum \frac{1}{3^n} = \sum \left(\frac{1}{3}\right)^n$$

$$\frac{1}{5+3^n} < \frac{1}{3^n}$$

convergent
geometric
series

so the given series
converges as well.

ex. Determine the convergence of $\sum_{n=1}^{\infty} \frac{1}{3n^2+2}$

$$\frac{1}{3n^2+2} < \frac{1}{3n^2} < \frac{1}{n^2}$$

$\sum \frac{1}{n^2}$
convergent p series

↑
Thus, this series converges, too

ex. Determine the convergence of $\sum_{n=1}^{\infty} \frac{5^n}{3n-2}$

$$\frac{5^n}{3n-2} > \frac{5^n}{3n}$$

these terms do not go to 0
by the n^{th} term test,
this series $\sum_{n=1}^{\infty} \frac{5^n}{3n}$ diverges

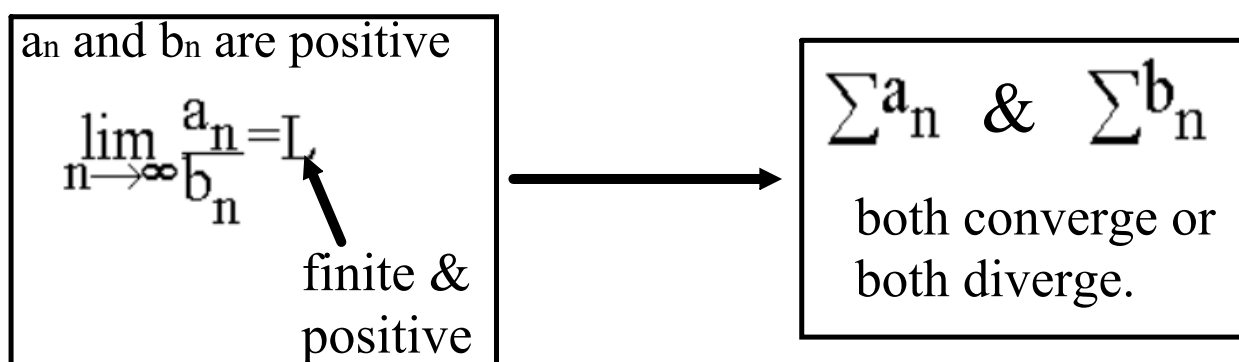
so, $\sum_{n=1}^{\infty} \frac{5^n}{3n-2}$ will also diverge.

ex. Determine the convergence of $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}-1}$

$$\frac{1}{\sqrt{n}-1} > \frac{1}{\sqrt{n}}$$

↑
p series with $p = \frac{1}{2} \leq 1$
so it diverges

By the direct comparison test
the given series also diverges.

Thm. Limit Comparison Test

Comparison series: keep only the highest powers of n in the numerator or denominator.

ex. Determine the convergence of $\sum_{n=1}^{\infty} \frac{2n^2-1}{3n^5+2n+1}$

compare to $\frac{n^2}{n^5} = \frac{1}{n^3}$

$$\lim_{n \rightarrow \infty} \frac{\frac{2n^2-1}{3n^5+2n+1} \cdot \frac{n^3}{1}}{\frac{1}{n^3}}$$

$$= \lim_{n \rightarrow \infty} \frac{2n^5 - n^3}{3n^5 + 2n + 1} = \frac{2}{3} \text{ which is finite \& positive}$$

so, $\sum \frac{1}{n^3}$ converges
and so does the given series.

ex. Determine the convergence of $\sum_{n=1}^{\infty} \frac{n+3}{n(n+4)}$

$$\lim_{n \rightarrow \infty} \frac{n+3}{n(n+4)} \cdot \frac{n}{1} = 1$$

$$\frac{n}{n^2} = \frac{1}{n}$$

↑
harmonic series

so the given series diverges,
just as the harmonic series diverges

ex. Determine the convergence of $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^2-1}}$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2-1}} \cdot \frac{\sqrt{n^2}}{1}$$

$$= \lim_{n \rightarrow \infty} \sqrt{\frac{n^2}{n^2-1}} = 1 \quad \text{so these series both diverge.}$$

$$\frac{1}{\sqrt{n^2}} = \frac{1}{n} \leftarrow \text{harmonic series}$$