9.5 Alternating Series Test

- $a_n > 0$
- $\lim_{n\to\infty} a_n = 0$
- $a_n \geq a_{n+1}$ for all n greater than some value

→

$$\sum_{n=1}^{\infty} (-1)^n a_n$$

&

$$\sum_{n=1}^{\infty} (-1)^{n+1} a_n$$

converge

ex. Determine converge for $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$

ex. Determine converge for $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n^2+1}$

Thm. Alternating Series Remainder

convergent alternating series

$a_n \geq a_{n+1}$
for all n greater
than some value

→

absolute value of the remainder R_n involved in approximating the sum S by S_n is no more than the first neglected term.
$|S-S_n| = |R_n| \leq a_{n+1}$.

ex. What is S_4 for this series? $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$

ex. How far off could we be by approximating by using the first 4 terms?

ex. Based on the above, the sum of this series must be between what 2 numbers?

Thm. <u>Absolute Convergence</u>

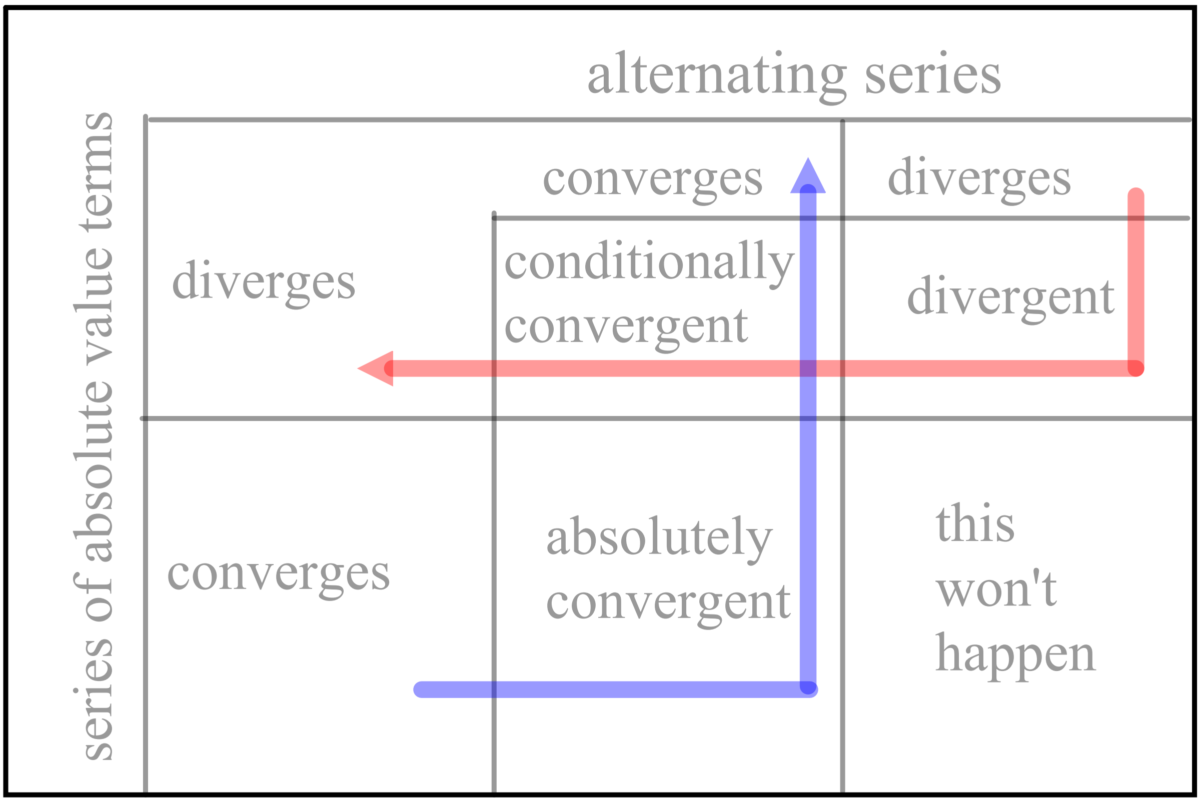
If a series of positive terms converges,
so does its alternating form.

Contrapositive: (If an alternating series diverges,
so does its absolute value form.)

		alternating series	
series of absolute value terms		converges	diverges
	diverges	conditionally convergent	divergent
	converges	absolutely convergent	this won't happen

ex. Classify each

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n} \qquad \sum_{n=1}^{\infty} 2\left(\frac{-1}{3}\right)^n \qquad \sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n^2+3}$$



One final strange fact (that isn't on the AP exam):

A conditionally convergent series could be rearranged to converge to *any* value one would want!