

9.6 The Ratio and Root Tests

For a series with nonzero terms,

Thm. The Ratio Test

$$\text{if } \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right|$$

Thm. The Root Test

$$\text{if } \lim_{n \rightarrow \infty} \sqrt[n]{a_n}$$

(not on syllabus, but can be used)

<1 , the series converges absolutely.

>1 or $=\infty$, the series diverges.

$=1$, the test is inconclusive, try a different test.

ex. Determine the convergence of $\sum_{n=0}^{\infty} \left(\frac{5^n}{(n+1)!} \right)$

ex. Determine the convergence of $\sum_{n=1}^{\infty} \frac{n3^n}{4^n}$

ex. Does the ratio test help us with $\sum_{n=1}^{\infty} \frac{1}{n^2}$?

ex. Determine the convergence of $\sum_{n=1}^{\infty} \left(\frac{2^{4n}}{n^n} \right)$

ex. Determine the convergence of $\sum_{n=1}^{\infty} \left(\frac{1}{[\ln(n+1)]^n} \right)$