9.6 The Ratio and Root Tests

For a series with nonzero terms,	
Thm. The Ratio Test if $\lim_{n\to\infty}\left\|\frac{a_{n+1}}{a_n}\right\|$	Thm. The Root Test if $\lim_{n\to\infty}\sqrt[n]{a_n}$ (not on syllabus, but can be used)
<1, the series converges absolutely. >1 or =∞, the series diverges. =1, the test is inconclusive, try a different test.	

ex. Determine the convergence of $\sum_{n=0}^{\infty}\left|\frac{5^n}{(n+1)!}\right|$

$$\lim_{n\to\infty}\left|\frac{\frac{5^{n+1}}{(n+1+1)!}}{\frac{5^n}{(n+1)!}}\right| = \lim_{n\to\infty}\frac{5^{n+1}}{(n+2)!}\cdot\frac{(n+1)!}{5^n}$$

$6\cdot5\cdot4\cdot3\cdot2\cdot1$

$7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1$

$$=\lim_{n\to\infty}\frac{5}{n+2}=0$$

the series converges absolutely

ex. Determine the convergence of $\sum_{n=1}^{\infty} \frac{n3^n}{4^n}$

$$\lim_{n\to\infty} \frac{(n+1)\,3^{\cancel{n+1}}}{4^{\cancel{n+1}}} \cdot \frac{\cancel{4^n}}{n\,\cancel{3^n}}$$

$$\lim_{n\to\infty} \frac{(n+1)\cdot 3}{n\cdot 4} = \frac{3}{4}$$

This converges absolutely.

ex.Does the ratio test help us with $\sum_{n=1}^{\infty} \frac{1}{n^2}$?

$$\lim_{n\to\infty} \frac{1}{(n+1)^2} \cdot \frac{n^2}{1}$$

Convergent
P series

$$\frac{1}{p-1}$$

$$\lim_{n\to\infty} \frac{n^2}{n^2+2n+1} = 1$$

No.

$$\frac{1}{2-1} = 1$$

ex. Determine the convergence of $\sum_{n=1}^{\infty}\left(\frac{2^{4n}}{n^n}\right)$

$$\lim_{n\to\infty} \sqrt[n]{\frac{2^{4n}}{n^n}} = \lim_{n\to\infty} \frac{2^4}{n} = 0$$

converges abs'ly

ex. Determine the convergence of $\sum_{n=1}^{\infty} \left(\frac{1}{[\ln(n+1)]^n} \right)$

$$\lim_{n\to\infty} \sqrt[n]{\frac{1}{[\ln(n+1)]^n}}$$

$$= \lim_{n\to\infty} \frac{1}{\ln(n+1)} = 0$$

converges abs'ly