

9.8 Power Series

Write a geometric series with x instead of r and you get:

Defn.
$$\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x^1 + a_2 x^2 + a_3 x^3 + \dots$$

is called a power series &

$$\sum_{n=0}^{\infty} a_n (x-c)^n = a_0 + a_1 (x-c)^1 + a_2 (x-c)^2 + a_3 (x-c)^3 + \dots$$

is called a power series centered at c for some constant c .

Remember:

$0 < |r| < 1$ brings convergence & $|r| \geq 1$ brings divergence
so, $0 < |x| < 1$ brings convergence & $|x| \geq 1$ brings divergence

ex. Based on what we did with geometric series, what does the power series

$$\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \dots$$

converge to?

Thm. Three cases:

A power series centered at c , either converges...

A. for c only (only at $x=c$).	B. absolutely for all x when $ x-c < R$, and diverges otherwise.	C. for all x .
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The number R is called the radius of convergence of the power series.

A. $R=0$.	B. R is some real number.	C. $R=\infty$
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The set of all x values for which the power series converges is called the interval of convergence.

To find the interval of convergence

1. Ratio test or root test finds interval of absolute convergence, usually $|x-c| < R$.
2. If case B, use some test for convergence at the endpoints.

ex. Determine the interval of convergence of $\sum_{n=0}^{\infty} \left(\frac{x^n}{n!} \right)$

ex. Determine the interval of convergence of $\sum_{n=0}^{\infty} n!x^n$

ex. Find the interval of convergence of $\sum_{n=1}^{\infty} (-1)^{n-1} \left(\frac{x^n}{n} \right)$

ex. Determine the interval of convergence of $\sum_{n=1}^{\infty} \frac{x^n}{n\sqrt{n}3^n}$

Thm. Power series can be integrated and differentiated term by term:

$$\text{If a function } f(x) = \sum_{n=0}^{\infty} a_n (x-c)^n$$

has a radius of convergence $R > 0$, then on $(c-R, c+R)$ f is continuous, differentiable, and integrable and in fact,

$$1. f'(x) = \sum_{n=0}^{\infty} n a_n (x-c)^{n-1} \text{ and}$$

$$2. \int f(x) dx = C + \sum_{n=0}^{\infty} \frac{a_n (x-c)^{n+1}}{n+1}$$

with all three series having the same R , but possibly different intervals due to behavior at the endpoints.

ex. Use the fact that

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} + \dots$$

converges for all x to

- Write the first 6 terms of a series for $\cos x$.
- Write a series for $\sin(2x)$.