

9.9 Representation of functions by power series

To get a geometric power series centered at c
(the Taylor series at c):

1. Separate $x-c$.
2. Divide everything by the constant term in the denominator to get a 1 in the denominator.
3. Identify a and r for the geometric series.

ex. Find a geometric power series centered at 0 for $f(x) = \frac{3}{2-x}$

ex. Find a geometric power series centered at 0 for $f(x) = \frac{3}{4-x}$
also find the interval of convergence.

Thm. Let $f(x) = \sum a_n x^n$ and $g(x) = \sum b_n x^n$ then

1. $f(kx) = \sum a_n k^n x^n$

2. $f(x^m) = \sum a_n x^{nm}$

3. $f(x) \pm g(x) = (a_n \pm b_n) x^n$

Note that these may change the interval of convergence.

ex. Find a geometric power series centered at 2 for $f(x) = \frac{3}{4-x}$

ex. For $\frac{1}{1+X} = \sum_{n=0}^{\infty} (-1)^n X^n$ Find the following:

$$\frac{1}{1-x}$$

$$\frac{1}{1+x^3}$$

$$-(1+x)^{-2}$$

$$\ln|1+x|$$