9.9 Representation of functions by power series

To get a geometric power series centered at c
(the Taylor series at c):
1. Separate x-c.
2. Divide everything by the constant term in the denominator to get a 1 in the denominator.
3. Identify a and r for the geometric series.

ex. Find a geometric power series centered at 0 for $f(x)=\frac{\frac{1}{2}\cdot 3}{\frac{1}{2}\cdot(2-x)}$

$$\frac{\frac{3}{2}}{\frac{2}{2}-\frac{x}{2}} = \frac{\boxed{\frac{3}{2}}}{1-\left(\frac{x}{2}\right)} \qquad \frac{\boxed{a}}{1-(r)}$$

$$\frac{3}{2-x} = \sum_{n=0}^{\infty} \frac{3}{2}\cdot\left(\frac{x}{2}\right)^n$$

ex. Find a geometric power series centered at 0 for f(x)= $\frac{3}{4-x}$ $\quad \frac{\cancel{4}\,1}{\frac{1}{4}}$

also find the interval of convergence.

$$\frac{3}{4-x} = \frac{\frac{3}{4}}{\frac{4}{4}-\frac{x}{4}} = \frac{\overset{a}{\frac{3}{4}}}{1-\underset{r}{\frac{x}{4}}} = \sum_{n=0}^{\infty} \frac{3}{4}\left(\frac{x}{4}\right)^n$$

$$\left|\frac{x}{4}\right| < 1$$

$$\frac{|x|}{4} < 1$$

$$|x| < 4$$

$$(-4, 4)$$

Thm. Let $f(x)= \sum a_n x^n$ and $g(x)= \sum b_n x^n$ then

1. $f(kx)= \sum a_n k^n x^n$
2. $f(x^m)= \sum a_n x^{nm}$
3. $f(x)\pm g(x)=(a_n \pm b_n)x^n$

Note that these may change the interval of convergence.

ex. Find a geometric power series centered at 2 for f(x)= $\frac{3}{4-x}$ $= \frac{3}{2-(x-2)}$

-2

$\frac{3}{6-(x+2)}$

$= \frac{\frac{3}{2}}{\frac{2}{2} - \frac{(x-2)}{2}}$

$\sum_{n=0}^{\infty} \frac{3}{2}\left(\frac{x-2}{2}\right)^n$

$r = \frac{x-2}{2}$

$a = \frac{3}{2}$

$\left|\frac{x-2}{2}\right| < 1$

$|x-2| < 2$

$-2 < x-2 < 2$
$+2 \quad +2 \quad +2$

$0 < x < 4 \qquad (0,4)$

ex. For $\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n$ Find the following:

$(1+x)^{-1}$ $|x|<1$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} (-1)^n(-x)^n = \sum_{n=0}^{\infty} \cancel{(-1)^n(-1)^n} x^n = \sum_{n=0}^{\infty} x^n$$

$$\frac{1}{1+x^3} = \sum_{n=0}^{\infty} (-1)^n (x^3)^n = \sum_{n=0}^{\infty} (-1)^n x^{3n}$$

$$-(1+x)^{-2} = \sum_{n=0}^{\infty} (-1)^n \cdot n \cdot x^{n-1}$$

$$\int 5(x) \rightarrow 5 \cdot \frac{x^2}{2}$$

$$\ln|1+x| = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1} + C$$