

9.9 Representation of functions by power series

To get a geometric power series centered at c
(the Taylor series at c):

1. Separate $x-c$.
2. Divide everything by the constant term in the denominator to get a 1 in the denominator.
3. Identify a and r for the geometric series.

ex. Find a geometric power series centered at 0 for $f(x) = \frac{3}{2-x}$

$$\frac{\frac{3}{2}}{\frac{2}{2} - \frac{x}{2}} = \frac{\boxed{\frac{3}{2}}}{1 - \boxed{\frac{x}{2}}} = \frac{\boxed{a}}{1 - r}$$

$$\frac{3}{2-x} = \sum_{n=0}^{\infty} \frac{3}{2} \cdot \left(\frac{x}{2}\right)^n$$

ex. Find a geometric power series centered at 0 for $f(x) = \frac{3}{4-x}$ also find the interval of convergence.

$$\frac{3}{4-x} = \frac{\frac{3}{4}}{\frac{4}{4} - \frac{x}{4}} = \frac{\frac{3}{4}}{1 - \frac{x}{4}} = \sum_{n=0}^{\infty} \frac{3}{4} \left(\frac{x}{4}\right)^n$$

$$\left|\frac{x}{4}\right| < 1$$

$$\frac{|x|}{4} < 1$$

$$|x| < 4$$

$$(-4, 4)$$

Thm. Let $f(x) = \sum a_n x^n$ and $g(x) = \sum b_n x^n$ then

1. $f(kx) = \sum a_n k^n x^n$

2. $f(x^m) = \sum a_n x^{nm}$

3. $f(x) \pm g(x) = (a_n \pm b_n) x^n$

Note that these may change the interval of convergence.

ex. Find a geometric power series centered at 2 for $f(x) = \frac{3}{4-x} = \frac{3}{2 - \underbrace{(x-2)}}_{}$

$$\frac{3}{6 - (x+2)}$$

$$= \frac{3}{2 - \frac{(x-2)}{2}}$$

$$\sum_{n=0}^{\infty} \frac{3}{2} \left(\frac{x-2}{2} \right)^n$$

$$r = \frac{x-2}{2}$$

$$a = \frac{3}{2}$$

$$\left| \frac{x-2}{2} \right| < 1$$

$$|x-2| < 2$$

$$\begin{array}{ccc} -2 < x-2 < 2 \\ +2 & +2 & +2 \end{array}$$

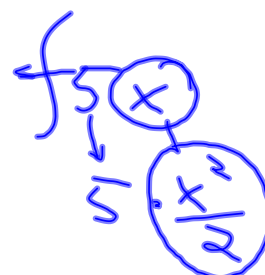
$$0 < x < 4 \quad (0, 4)$$

ex. For $\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n$ Find the following:
 $(1+x)^{-1}$ $|x| < 1$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} (-1)^n (-x)^n = \sum_{n=0}^{\infty} \cancel{(-1)^n} \cancel{(-1)^n} x^n = \sum_{n=0}^{\infty} x^n$$

$$\frac{1}{1+x^3} = \sum_{n=0}^{\infty} (-1)^n (x^3)^n = \sum_{n=0}^{\infty} (-1)^n x^{3n}$$

$$-(1+x)^{-2} = \sum_{n=0}^{\infty} (-1)^n \cdot n \cdot x^{n-1}$$



$$\ln|1+x| = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1} + C$$