

## 12.1 Inference for a population proportion

I think that 50% of the cm cubes in your bag is yellow.

Test my claim-- is there evidence that the proportion is 0.5?

We wonder about how much of a **population** has **some characteristic**.

Take an SRS of size  $n$  from the **population** and count the #  $X$  of **successes**.

The sample proportion of **successes**

$$\hat{p} = \frac{X}{n}$$

estimates the unknown population proportion  $p$ .

What % of **FHN students** have an **iPhone**.

SRS of  $n$  **FHN students**

Find the sample proportion of **successes**

$$\hat{p} = \frac{X}{n}$$

estimates the unknown proportion.

(page 686)

Remember? For the sampling distribution of  $\hat{p}$

$$\mu_{\hat{p}} = p \quad \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

We don't know the parameter  $p$ ,  
so we estimate with  $\hat{p}$ :  $SE_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

(We do something similar in ch. 11 with  $s$  for  $\sigma$ .)

Conditions/Assumptions (page 687)

**random**

**independent**

**ten %**

**and large enough  $n$**  ←

**this lets us  
use z scores  
and normal  
approximations**

more detail on the next slide



Conditions/Assumptions for inference about a proportion (page 687)

- random sample
- of independent chosen items
- with less than ten % of the population ( $N > 10n$ ), but
- a large enough  $n$  that we see (CI) or expect (HT/TOS) at least 10 successes and at least 10 failures

$$\underline{n\hat{p} \geq 10} \quad \underline{n(1 - \hat{p}) \geq 10}$$

$$\underline{np_0 \geq 10} \quad \underline{n(1 - p_0) \geq 10}$$

In a CI, we don't propose  $p$ , so we need to see  $\geq 10$  successes and  $\geq 10$  failures

In  $H_0: p=p_0$ , we propose  $p$ , so we need to expect  $\geq 10$  successes and  $\geq 10$  failures

Very important distinction for later:

For CI use

$$\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

In a CI, we don't propose  $p$ , so we use the observed proportion

For HT/TOS use

$$\sqrt{\frac{p_0(1-p_0)}{n}}$$

In  $H_0: p=p_0$ , we propose  $p$ , so we use the hypothesized proportion

As a result, there is no exact correspondence between the CI and two-sided test.

## Confidence Interval for a Population Proportion (page 689)

An approximate level C confidence interval for p is

*categorical*  
*quantitative*  
*z ap ta x*

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

remember,  $z^*$  is the upper  $(1-C)/2$  standard normal critical value.

Example:

In a random sample of 200 students applying to universities in some state, 10% didn't meet the proposed new standards for math proficiency. What is the 90% CI for the proportion of all college applicants in the state who didn't meet standards?



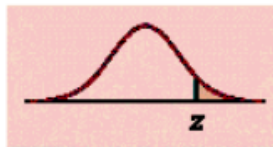
## Significance test for a population proportion $p$ (page 689)

To test the hypothesis  $H_0: p = p_0$ , compute the  $z$  statistic

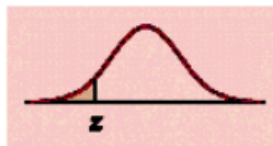
$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

For variable  $Z$  having  $N(0,1)$ , the approximate P-value for a test of  $H_0$  against

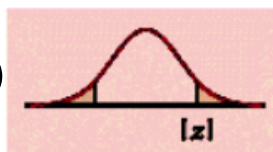
$H_a: p > p_0$  is  $P(Z \geq z)$



$H_a: p < p_0$  is  $P(Z \leq z)$



$H_a: p \neq p_0$  is  $2P(Z \geq |z|)$



| z   | .00   | .01   | .02   | .03   | .04   | .05   | .06   | .07   | .08   | .09   |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 0.0 | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 | .5319 | .5359 |
| 0.1 | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5636 | .5675 | .5714 | .5753 |
| 0.2 | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 | .6103 | .6141 |
| 0.3 | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 | .6480 | .6517 |
| 0.4 | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 | .6844 | .6879 |
| 0.5 | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 | .7190 | .7224 |
| 0.6 | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 | .7517 | .7549 |
| 0.7 | .7580 | .7611 | .7642 | .7673 | .7704 | .7734 | .7764 | .7794 | .7823 | .7852 |
| 0.8 | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 | .8106 | .8133 |
| 0.9 | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 | .8365 | .8389 |
| 1.0 | .8413 | .8438 | .8461 | .8485 | .8508 | .8531 | .8554 | .8577 | .8599 | .8621 |
| 1.1 | .8643 | .8665 | .8686 | .8708 | .8729 | .8749 | .8770 | .8790 | .8810 | .8830 |
| 1.2 | .8849 | .8869 | .8888 | .8907 | .8925 | .8944 | .8962 | .8980 | .8997 | .9015 |
| 1.3 | .9032 | .9049 | .9066 | .9082 | .9099 | .9115 | .9131 | .9147 | .9162 | .9177 |
| 1.4 | .9192 | .9207 | .9222 | .9236 | .9251 | .9265 | .9279 | .9292 | .9306 | .9319 |
| 1.5 | .9332 | .9345 | .9357 | .9370 | .9382 | .9394 | .9406 | .9418 | .9429 | .9441 |
| 1.6 | .9452 | .9463 | .9474 | .9484 | .9495 | .9505 | .9515 | .9525 | .9535 | .9545 |
| 1.7 | .9554 | .9564 | .9573 | .9582 | .9591 | .9599 | .9608 | .9616 | .9625 | .9633 |
| 1.8 | .9641 | .9649 | .9656 | .9664 | .9671 | .9678 | .9686 | .9693 | .9699 | .9706 |
| 1.9 | .9713 | .9719 | .9726 | .9732 | .9738 | .9744 | .9750 | .9756 | .9761 | .9767 |
| 2.0 | .9772 | .9778 | .9783 | .9788 | .9793 | .9798 | .9803 | .9808 | .9812 | .9817 |
| 2.1 | .9821 | .9826 | .9830 | .9834 | .9838 | .9842 | .9846 | .9850 | .9854 | .9857 |
| 2.2 | .9861 | .9864 | .9868 | .9871 | .9875 | .9878 | .9881 | .9884 | .9887 | .9890 |
| 2.3 | .9893 | .9896 | .9898 | .9901 | .9904 | .9906 | .9909 | .9911 | .9913 | .9916 |
| 2.4 | .9918 | .9920 | .9922 | .9925 | .9927 | .9929 | .9931 | .9932 | .9934 | .9936 |
| 2.5 | .9938 | .9940 | .9941 | .9943 | .9945 | .9946 | .9948 | .9949 | .9951 | .9952 |
| 2.6 | .9953 | .9955 | .9956 | .9957 | .9959 | .9960 | .9961 | .9962 | .9963 | .9964 |
| 2.7 | .9965 | .9966 | .9967 | .9968 | .9969 | .9970 | .9971 | .9972 | .9973 | .9974 |
| 2.8 | .9974 | .9975 | .9976 | .9977 | .9977 | .9978 | .9979 | .9979 | .9980 | .9981 |
| 2.9 | .9981 | .9982 | .9982 | .9983 | .9984 | .9984 | .9985 | .9985 | .9986 | .9986 |
| 3.0 | .9987 | .9987 | .9987 | .9988 | .9988 | .9989 | .9989 | .9989 | .9990 | .9990 |
| 3.1 | .9990 | .9991 | .9991 | .9991 | .9992 | .9992 | .9992 | .9992 | .9993 | .9993 |
| 3.2 | .9993 | .9993 | .9994 | .9994 | .9994 | .9994 | .9994 | .9995 | .9995 | .9995 |
| 3.3 | .9995 | .9995 | .9995 | .9996 | .9996 | .9996 | .9996 | .9996 | .9996 | .9997 |
| 3.4 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9998 |

Choosing a sample size (page 695)

**To determine the sample size n** that gives a confidence interval with confidence level C with a specified margin of error ME, solve this for n:

$$ME \geq z^* \sqrt{\frac{p^*(1-p^*)}{n}}$$

where  $p^*$  is the most recent value of  $\hat{p}$ , a past estimate for the  $p$ .

When we don't have a value of  $\hat{p}$ , we use 0.5 instead.

an example coming up later



Why use a guess like 0.5 for  $p^*$ ?

To make the most cautious estimate when we have no prior information about the likely value of  $p$ .

| $p^*$     | $(1-p^*)$ | $p^*(1-p^*)$ |
|-----------|-----------|--------------|
| .1        | .9        | .09          |
| .2        | .8        | .16          |
| .3        | .7        | .21          |
| .4        | .6        | .24          |
| <b>.5</b> | <b>.5</b> | <b>.25</b>   |
| .6        | .4        | .24          |
| .7        | .3        | .21          |
| .8        | .2        | .16          |
| .9        | .1        | .09          |



When  $p^*(1-p^*)$  is 0.25, then  $n$  is as large as possible, for the given confidence level and margin of error.

example:

What sample size is needed to get a 99% CI with margin of error less than or equal to .02?

$$ME = z^* \sqrt{\frac{p(1-p)}{n}}$$

example:

What n is needed above if a previous study had  $\hat{p} = 0.37$ ?

| $z$  | .00   | .01   | .02   | .03   | .04   | .05   | .06   | .07   | .08   | .09   |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| -3.4 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0002 |
| -3.3 | .0005 | .0005 | .0005 | .0004 | .0004 | .0004 | .0004 | .0004 | .0004 | .0003 |
| -3.2 | .0007 | .0007 | .0006 | .0006 | .0006 | .0006 | .0006 | .0005 | .0005 | .0005 |
| -3.1 | .0010 | .0009 | .0009 | .0009 | .0008 | .0008 | .0008 | .0008 | .0007 | .0007 |
| -3.0 | .0013 | .0013 | .0013 | .0012 | .0012 | .0011 | .0011 | .0011 | .0010 | .0010 |
| -2.9 | .0019 | .0018 | .0018 | .0017 | .0016 | .0016 | .0015 | .0015 | .0014 | .0014 |
| -2.8 | .0026 | .0025 | .0024 | .0023 | .0023 | .0022 | .0021 | .0021 | .0020 | .0019 |
| -2.7 | .0035 | .0034 | .0033 | .0032 | .0031 | .0030 | .0029 | .0028 | .0027 | .0026 |
| -2.6 | .0047 | .0045 | .0044 | .0043 | .0041 | .0040 | .0039 | .0038 | .0037 | .0036 |
| -2.5 | .0062 | .0060 | .0059 | .0057 | .0055 | .0054 | .0052 | .0051 | .0049 | .0048 |
| -2.4 | .0082 | .0080 | .0078 | .0075 | .0073 | .0071 | .0069 | .0068 | .0066 | .0064 |
| -2.3 | .0107 | .0104 | .0102 | .0099 | .0096 | .0094 | .0091 | .0089 | .0087 | .0084 |
| -2.2 | .0139 | .0136 | .0132 | .0129 | .0125 | .0122 | .0119 | .0116 | .0113 | .0110 |
| -2.1 | .0179 | .0174 | .0170 | .0166 | .0162 | .0158 | .0154 | .0150 | .0146 | .0143 |
| -2.0 | .0228 | .0222 | .0217 | .0212 | .0207 | .0202 | .0197 | .0192 | .0188 | .0183 |
| -1.9 | .0287 | .0281 | .0274 | .0268 | .0262 | .0256 | .0250 | .0244 | .0239 | .0233 |
| -1.8 | .0359 | .0351 | .0344 | .0336 | .0329 | .0322 | .0314 | .0307 | .0301 | .0294 |
| -1.7 | .0446 | .0436 | .0427 | .0418 | .0409 | .0401 | .0392 | .0384 | .0375 | .0367 |
| -1.6 | .0548 | .0537 | .0526 | .0516 | .0505 | .0495 | .0485 | .0475 | .0465 | .0455 |
| -1.5 | .0668 | .0655 | .0643 | .0630 | .0618 | .0606 | .0594 | .0582 | .0571 | .0559 |
| -1.4 | .0808 | .0793 | .0778 | .0764 | .0749 | .0735 | .0721 | .0708 | .0694 | .0681 |
| -1.3 | .0968 | .0951 | .0934 | .0918 | .0901 | .0885 | .0869 | .0853 | .0838 | .0823 |
| -1.2 | .1151 | .1131 | .1112 | .1093 | .1075 | .1056 | .1038 | .1020 | .1003 | .0985 |
| -1.1 | .1357 | .1335 | .1314 | .1292 | .1271 | .1251 | .1230 | .1210 | .1190 | .1170 |
| -1.0 | .1587 | .1562 | .1539 | .1515 | .1492 | .1469 | .1446 | .1423 | .1401 | .1379 |
| -0.9 | .1841 | .1814 | .1788 | .1762 | .1736 | .1711 | .1685 | .1660 | .1635 | .1611 |
| -0.8 | .2119 | .2090 | .2061 | .2033 | .2005 | .1977 | .1949 | .1922 | .1894 | .1867 |
| -0.7 | .2420 | .2389 | .2358 | .2327 | .2296 | .2266 | .2236 | .2206 | .2177 | .2148 |
| -0.6 | .2743 | .2709 | .2676 | .2643 | .2611 | .2578 | .2546 | .2514 | .2483 | .2451 |
| -0.5 | .3085 | .3050 | .3015 | .2981 | .2946 | .2912 | .2877 | .2843 | .2810 | .2776 |
| -0.4 | .3446 | .3409 | .3372 | .3336 | .3300 | .3264 | .3228 | .3192 | .3156 | .3121 |
| -0.3 | .3821 | .3783 | .3745 | .3707 | .3669 | .3632 | .3594 | .3557 | .3520 | .3483 |
| -0.2 | .4207 | .4168 | .4129 | .4090 | .4052 | .4013 | .3974 | .3936 | .3897 | .3859 |
| -0.1 | .4602 | .4562 | .4522 | .4483 | .4443 | .4404 | .4364 | .4325 | .4286 | .4247 |
| -0.0 | .5000 | .4960 | .4920 | .4880 | .4840 | .4801 | .4761 | .4721 | .4681 | .4641 |

**Table B** *t* distribution critical values

| df   | Tail probability <i>p</i> |      |       |       |       |       |       |       |       |       |       |       |
|------|---------------------------|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
|      | .25                       | .20  | .15   | .10   | .05   | .025  | .02   | .01   | .005  | .0025 | .001  | .0005 |
| 60   | .679                      | .848 | 1.045 | 1.296 | 1.671 | 2.000 | 2.099 | 2.390 | 2.660 | 2.915 | 3.232 | 3.460 |
| 80   | .678                      | .846 | 1.043 | 1.292 | 1.664 | 1.990 | 2.088 | 2.374 | 2.639 | 2.887 | 3.195 | 3.416 |
| 100  | .677                      | .845 | 1.042 | 1.290 | 1.660 | 1.984 | 2.081 | 2.364 | 2.626 | 2.871 | 3.174 | 3.390 |
| 1000 | .675                      | .842 | 1.037 | 1.282 | 1.646 | 1.962 | 2.056 | 2.330 | 2.581 | 2.813 | 3.098 | 3.300 |
| ∞    | .674                      | .841 | 1.036 | 1.282 | 1.645 | 1.960 | 2.054 | 2.326 | 2.576 | 2.807 | 3.091 | 3.291 |
|      | 50%                       | 60%  | 70%   | 80%   | 90%   | 95%   | 96%   | 98%   | 99%   | 99.5% | 99.8% | 99.9% |

Confidence level *C*

From the *New York Times*:

Point estimate in headline:

"40% in Survey Say Inflation is Major Issue"

Confidence interval wording later in the article:

"In theory, one can say with 95 percent certainty that the results based on the entire sample differ by no more than 3 percentage points in either direction from what would have been obtained by interviewing all adult Americans."

Confidence interval as expressed in [statistics](#):

"We are 95% confident that the actual percent of adult Americans who say inflation is a major issue is between 37% and 43%."