13.1 Test for Goodness of Fit

color	observed count O	hypothesized proportion	expected count E	component $\frac{(O-E)^2}{E}$
white	144			
pink	216			
orange	184			
yellow	187			
green	88			
purple	202			

Table C

χ^2 critical values

df	Tail probability p										
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4

Tom's might buy a small gas station and wants to plan for gasoline sales throughout the year. He asks the seller for sales data. A SRS shows 18,448 gallons of gas sold last year. Tom believes sales would be fairly consistent from month to month, in fact, essentially uniform. How could we test this belief?

Q: If sales **ARE** uniform and total 18,448 gallons, what would we expect of these sales?

A: $\frac{\text{18448 gallons}}{\text{12 months}}$ =1537.3 gallons/month

We would expect sales of 1537.333 gallons of gas per month.

If sales really **<u>ARE</u>** uniformly distributed, how unusual are these data?

Month	Gallons
January	1,610
February	1,581
March	1,649
April	1,590
May	1,540
June	1,397
July	1,416
August	1,350
September	1,495
October	1,564
November	1,601
December	1,655
	18,448

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A **goodness of fit test** helps us decide whether a population has some distribution categories.

Suppose a hypothesized distribution has n outcome categories. To test the hypothesis

H_o: the actual population proportions are equal to the hypothesized proportions

first calculate the chi-square test statistic: $X^2 = \sum \frac{(O-E)^2}{E}$

which has approximately a chi-square distribution with (n-1) degrees of freedom.

For a test of H_o against the alternative hypothesis,

H_a: the actual population proportions are *different from* the hypothesized proportions,

the P-value is $P(\chi^2 \geq X^2)$

Conditions:

- Random
- Independent (Ten % or less)
- Large enough n that all individual expected counts are at least 1 & no more that 1/5 of expected counts are under 5

The inference toolbox page 571

Step 1: ID the pop., parameter referenced, H_o and H_a:

Step 2: Choose the appropriate inference procedure and verify the conditions:

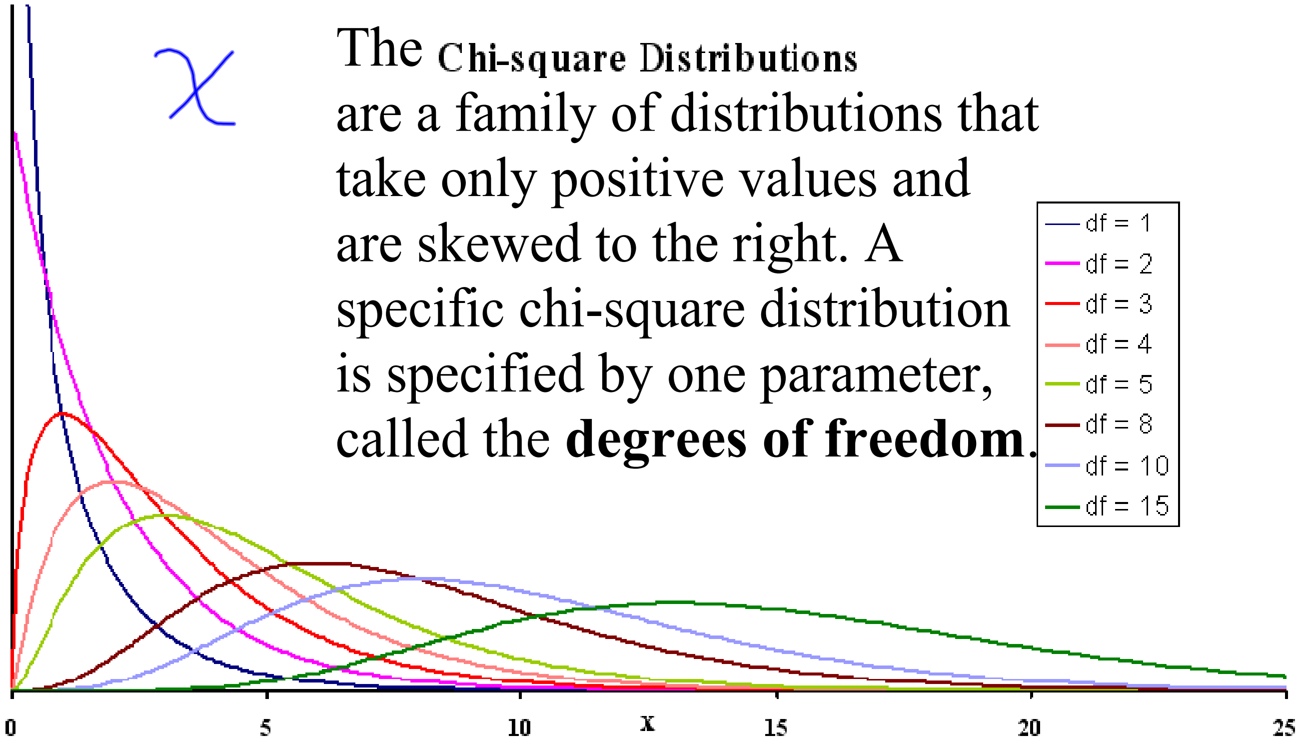
Step 3: If conditions met, carry out inference procedure:

Month	Observed	Expected
January	1610	1537.333
February	1581	1537.333
March	1649	1537.333
April	1590	1537.333
May	1540	1537.333
June	1397	1537.333
July	1416	1537.333
August	1350	1537.333
September	1495	1537.333
October	1564	1537.333
November	1601	1537.333
December	1655	1537.333
	18448	18448

Step 4: Interpret the results in the context of the problem.

The Chi-Square Distributions pg. 730

The **Chi-square Distributions** are a family of distributions that take only positive values and are skewed to the right. A specific chi-square distribution is specified by one parameter, called the **degrees of freedom**.



Lion #39: Stalking Role Distribution

A	B	C	D	E	F	G
0	9	15	56	54	10	1