13.2 Inference for Two-Way Tables

A two-way table gives counts for successes and failures or for counts that compare two categorical variables.

Observed Frequencies				
	Chocolate Preference			
Gender	White	Milk	Dark	**Total**
Male	20	40	20	80
Female	30	30	10	70
Total	50	70	30	150

Two new tests:

Test for Homogeneity

- If experimental units come from **different populations**, then we want to know whether the category (column) proportions are the same for each population - this is a test for homogeneity.

Test for Independence

- If experimental units come from the **same population**, then we want to know whether the row and column variables are independent - this is a test for independence.

Test of Homogeneity

Prospective:

Two distinct, different, independent groups

Goal: Determine if proportions are the same for each population.

	sore throat	no sore throat	totals
placebo	289	71	360
zinc lozenge	302	105	407
totals	591	176	767

H_o: The distribution of the response variable is the same in all populations.

Test of Independence

Retrospective:

A single group

Goal: Determine if there's a relationship between two variables.

	hemophiliac	non hemophiliac	totals
male	53	260	313
female	14	373	387
totals	67	633	700

H_o: There is no relationship between the two categorical variables.

A rule of thumb that works with AP-level statistics.

Ask "how did I sample and what variable(s) did I keep track of"?

From one population and
distinguished categorical values on one variable → goodness of fit

From one population and
distinguished categorical values on two variables → test of independence

From more than one population and
distinguished values on one variable → homogeneity of proportions

pg. 747

The **expected count** in any cell of a two-way table when H_o is true is:

$$\text{expected count} = \frac{(\text{row total})(\text{column total})}{\text{table total}}$$

example:

expected number of males that prefer white chocolate $\dfrac{(80)(50)}{(150)} = 26.67$

pg. 750

The **chi-square statistic** is a measure of how far the observed counts in a two-way table are from the expected counts. The formula for the statistic is

$$\chi^2 = \sum \left[\frac{(\text{observed count - expected count})^2}{\text{expected count}} \right]$$

The sum is over all r × c cells in the table.

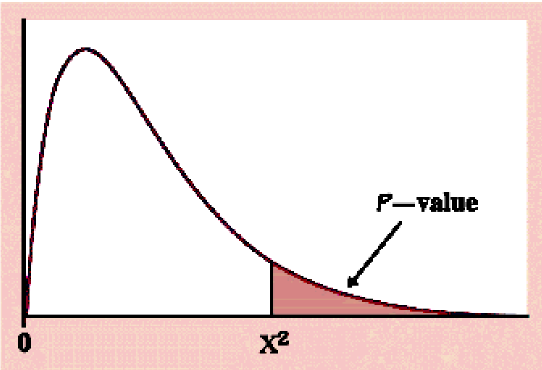
Test for Homogeneity pg. 751

Select independent SRSs from each of c populations. Classify each individual in a sample according to a categorical response variable with r possible values. There are c different sets of proportions to be compared, one for each population.

H_o: the distribution of the response variable is the same in all c populations. H_a: these c distributions are not all the same.

If H_o is true, the chi-square statistic has approximately a chi-square distribution with (r-1)(c-1) degrees of freedom (df).

The P-value for the chi square test is the area to the right of the chi-square statistic under the chi-square density curve with df degrees of freedom.



pg. 752
You can safely use the chi-square test with critical values from the chi-square distribution when no more than 20% of the expected counts are less than 5 and all individual expected counts are 1 or greater.

In particular, all four expected counts in a 2 × 2 table should be 5 or greater.

example: Medical researchers randomly assigned 767 mer to two experimental groups (placebo or zinc lozenge), ther innoculated each with a strain of the common cold. The following table gives the results.

	sore throat	no sore throat	totals
placebo	289	71	360
zinc lozenge	302	105	407
totals	591	176	767

Test of Independence pg. 758
Use to test the null hypothesis:

H_o: there is no relationship between two categorical variables

when you have a two-way table from a single SRS, with each individual classified according to both of two categorical variables.

A candy company randomly surveys 150 adults in a large city recording their preference for certain types of chocolate and their gender. The table below indicates the responses.

Observed Frequencies				
	Chocolate Preference			
Gender	White	Milk	Dark	**Total**
Male	20	40	20	80
Female	30	30	10	70
Total	50	70	30	150

Which test to use (homogeneous or independent)?

1. Is high blood pressure dangerous? Medical researchers classified each of a group of men as "high" or "low" blood pressure, then watched them for 5 years. (a table would be given, but is not repeated here)

The wording seems to suggest that all subjects were from the same original group. If so, the question would be whether there is evidence of an association between blood pressure level and mortality--**a test of independence**.

Which test to use (homogeneous or independent)?

2. Market researchers know that background music can influence the mood and purchasing behavior of customers. Over a period of a year, one study in a supermarket in Northern Ireland compared three treatments of music: none, French accordian, and Italian string. Under each condition, the researchers recorded the numbers of French, Italian, and other wine purchased. (a table would be given, but is not repeated here)

Over a long period (a year), we have played music of various types, thereby setting up three populations of shoppers. We the counted values of one variable (sales). This means we apply **a test of homogeneity**. three separate groups of shoppers--one for each type of music played as shopping was taking place. We test whether each group behaved essentially like the others in th choice of wine to purchase.

Table 1: Sample of writing of Aristotle (384 - 322 BCE)

Work	Sentences w/ gar (n)	Sentences w/o gar (n)
Parts of Animals	63	137
Progression of Animals	58	142
De Caelo Book 1	84	216
De Caelo Book 2	103	197
Categoriae	25	75
De Interpetatione	72	228
Eudemian Ethics Book 1	45	101

Distribution of age class and stranding

Stranding	Young	Intermediate	Old
Stranded	41	125	52
Not stranded	153	364	70