6.2 Probability Models

There was a statistician who, when driving his car, would always accelerate hard before coming to an intersection, whiz straight through it, and slow down again once he was beyond it. One day, a passenger, understandably very unnerved by his driving style, asked the statistician why he would cross intersections so rapidly. The statistician replied:

"Well, statistically speaking, you are for more likely to have an accident at an intersection, so I just make sure that I spend less time there."

sample space S
set of all possible outcomes of a random phenomenon

event
outcome or set of outcomes of a random phenomenon.

event: tossing one coin
sample space: 2 possible outcomes
(heads or tails)

event: roll one die
sample space: 6 possible outcomes
(1, 2, 3, 4, 5, 6)

If a menu offers 5 choices for main dish, 5 for side, 5 for beverage, how many combinations are possible?

Multiplication Principle

1 thing happens m ways & another thing happens n ways, so together they both happen mn ways.

How many different large 1-topping pizzas could be made from 3 available crusts and 14 available toppings?

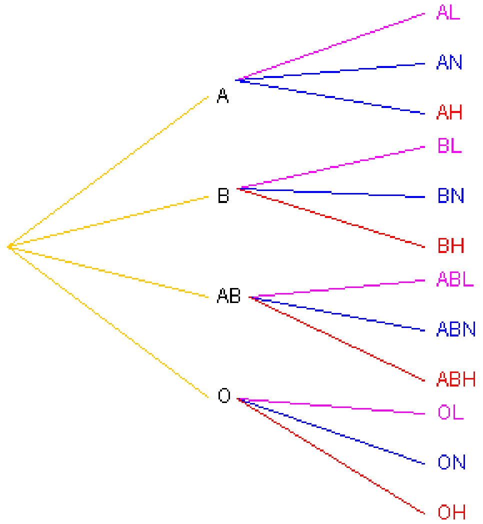
14*3=42

If there are 4 sizes, how many different 1-topping pizzas could be made?

14*3*4=168

Tree diagrams with probability

4 blood types   3 blood pressure groups



Sampling	**with replacement:**	**without replacement:**
What it is: selecting an item and then...	putting it back	leaving it out
Is each unit replaced before the next is sampled?	Yes	No
Impact on outcomes?	One outcome does not affect the other outcomes.	Each outcome depends on all previous outcomes.
Outcomes independent?	No, not independent	Yes, independent
example:	Take a card from a deck, put it back, take a 2nd card, put it back, take a 3rd...	Take a card from a deck, take a 2nd card, take a 3rd...

Probability Rules

1. Remember that $0 \leq P(A) \leq 1$ always
2. If S denotes the sample space, then $P(S) = 1$
3. $P(A') = 1 - P(A)$ for every event A where A' is the complement of A
4. Addition rule for disjoint (mutually exclusive) events: $P(A \text{ or } B) = P(A) + P(B)$

S is the set of all outcomes so P(S) is the sum of all probabilities for the outcomes.

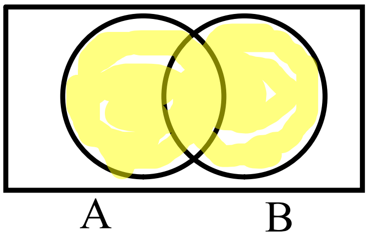
It's good to remember that your book uses A^c for the complement and some books use $\bar{A}$.

ActivStats III 13-2 Addition Rule

A Union B

all the elements that are in A or in B (or both)

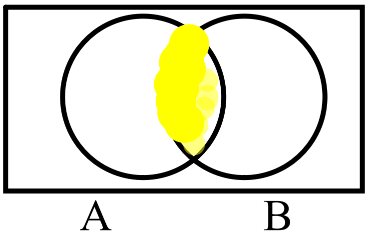
$A \cup B$



A Intersect B

all the elements that are in A <u>and</u> in B

$A \cap B$



For equally likely outcomes, the probability

$$P(A) = \frac{\text{number of outcomes that are A}}{\text{total number of outcomes in S}}$$

5. Multiplication rule for independent events:
P(A and B) = P(A)P(B)

Two events are independent if knowing that one occurs does not change the probability that the other occurs.

Venn Diagrams can tell us whether events are disjoint, but they can't tell us whether events are independent.

ActivStats III 13-2 Multiplication Rule

"Dumb Dora" read that there is about a one in one million chance that someone will board an airplane carrying a bomb, so she started carrying a bomb with her on every flight she takes. The way she figure it, the probability of two people having a bomb on the same plane are 1 in a trillion.

A total of 4000 cans are opened around the world every second. Ten babies are conceived around the world every second. Therefore, each time you open a can, you stand a 1 in 400 chance of becoming pregnant.

☺

Following pages are just more examples.

ODD and EVEN <u>are</u> mutually exclusive events
on <u>one</u> roll of a die.

ODD on the first and EVEN on the second <u>are not</u> mutually exclusive events on <u>two</u> rolls of the die.

On <u>one</u> roll, knowing the roll was ODD tells you the roll can't be EVEN, so they <u>aren't</u> independent.

On <u>two</u> rolls, rolling ODD the first time does not affect P(EVEN on the second roll) so they <u>are</u> independent. Since both events can happen (ODD on roll 1, EVEN on roll 2) they are also not mutually exclusive (not disjoint).

For a standard deck of cards, let events
D = diamonds and G = green.
Notice that: P(G | D) = P(G), because both are 0.
The fact that the card drawn was a diamond did not change the probability that the card was green, so they are independent. There are no green diamonds in the deck, so they are also disjoint.

Under any other circumstances, disjoint events cannot be independent; knowing that one has happened tells you immediately that the other cannot. Using D = diamonds again and B = black, we see they are again disjoint. However, P(B | D) = 0 and P(B) = 1/2, so they are not independent. Knowing that the card drawn is a diamond gives you a big hint about whether it might be black; these events cannot be independent because they are disjoint.

Independent events don't influence each other;
disjoint events can't both happen.

* Toss a coin with one hand and roll a die with the other. Getting a Head and a 6 are certainly independent - the coin doe not influence the die. They're not disjoint - both can happen (w expect about 1 time in 12). Independent, not disjoint.

* Pick a card. You can't get a red spade, so Red and Spade are disjoint. And that means they are not independent. As soon as you know the card is red, you know it cannot be a spade. Red precludes Spade. Disjoint, not independent.

* Think about going to a baseball game when there's the possibility of rain. If it's only a light rain, the game might go on rain and baseball are not disjoint. But they are not independent either - as soon as it starts raining the probability that the game will be played drops. Not disjoint, and not independent either.

The question "Where did you have lunch?" is asked and various probabilities can be imagined:

P(Taco Bell)
P(Chili's)
P(Hardee's)
P(Quizno's)
etc.

Of course, these events (even for the hungriest individuals!) are mutually exclusive (disjoint); if you know someone had lunch at Taco Bell you know they didn't go to Hardee's. So they can't be independent.

Regarding independence -- Let's assume Hardee's generally appeals more to males, so P(Hardee's| male) does not equal P (Hardee's), so gender and eating at Hardee's are not independent. We'll also assume Taco Bell has an appeal for both, so we could at least imagine that P(Taco Bell | male) = P(Taco Bell | female) = P(Taco Bell), so gender and eating at Taco Bell are independent. Thus we have gender and Hardees being not independent and not disjoint while we have gender and Taco Bell being independent and not disjoint.

A=rolling a 1 on a die
B=rolling a 2 on a die
$P(A)=1/6$
$P(B)=1/6$
$P(A\&B)=0$ so disjoint
$P(A|B)=0$ while $P(A)=1/6$, so not independent