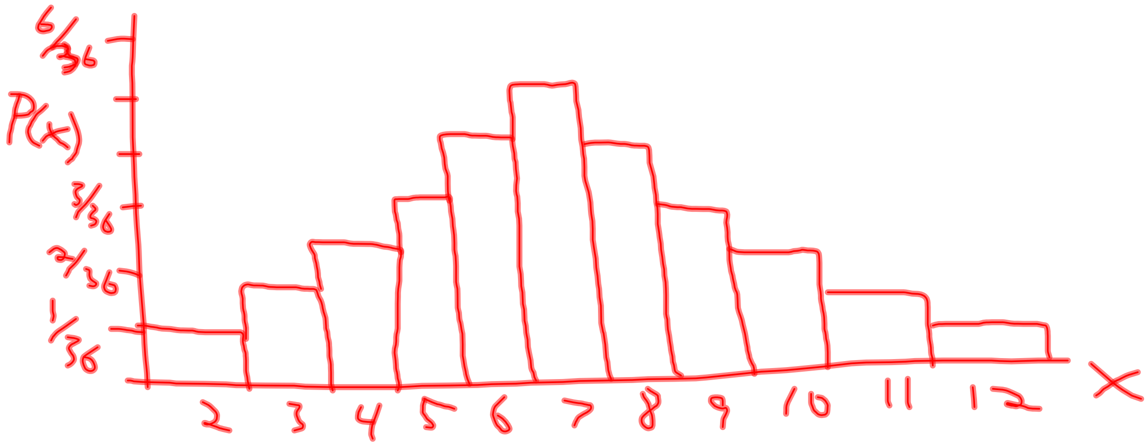
1. Convert between a graph and table to display a probability distribution or density curve.
I roll 2 different 6-sided dice. Let X be the sum of the dice. Make a probability distribution for the discrete random variable X. Then make a probability histogram for X.

X	2	3	4	5	6	7	8	9	10	11	12
P(x)	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$



2. Calculate probabilities for values of a random variable when given the probability distribution for the random variable or when given a density curve.

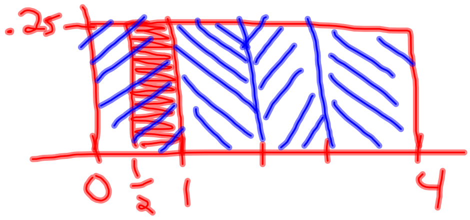
For X as defined above, what is the probability of rolling a total less than 5?

$$P(X<5) = \frac{1}{36} + \frac{2}{36} + \frac{3}{36} = \frac{6}{36}$$

For X as defined above, what is the probability of rolling a total of 8, 9, or 10?

$$P(8, 9, \text{ or } 10) = \frac{5}{36} + \frac{4}{36} + \frac{3}{36} = \frac{12}{36}$$

The probability density curve of a random variable W is a rectangle with base 0 to 4 and height 0.25, what is the probability that W is between .5 and 1?



$$P(.5 < w < 1) = \left(\frac{1}{2}\right)(.25) = \frac{1}{8}$$

3. Calculate the mean or expected value for a random variable when given the probability distribution for the random variable.

What is the mean for X as defined above?

$$\mu_x = E(x) = 7 = 2\left(\frac{1}{36}\right) + 3\left(\frac{2}{36}\right) + \cdots + 12\left(\frac{1}{36}\right)$$

What is the mean for Y, the outcome when a single 6-sided die is rolled?

$$\mu_Y = E(y) = 3.5 = 1\left(\frac{1}{6}\right) + 2\left(\frac{1}{6}\right) + 3\left(\frac{1}{6}\right) + \cdots + 6\left(\frac{1}{6}\right)$$

What is the mean for Z, the outcome when a single 4-sided die is rolled?

$$\mu_z = E(z) = 2.5 = 1(.25) + 2(.25) + \cdots 4(.25)$$

4. Calculate the standard deviation for a random variable when given the probability distribution for the random variable.
What is the standard deviation for X as defined above?

$\sigma_X = 2.415$

What is the standard deviation for Y, the outcome when a single 6-sided die is rolled?

$\sigma_Y = 1.708$

What is the standard deviation for Z, the outcome when a single 4-sided die is rolled?

$$\sigma_Z = 1.112 = \sqrt{(1-2.5)^2(.25)+(2-2.5)^2(.25)+(3-2.5)^2(.25)+(4-2.5)^2(.25)}$$

5. Calculate a probability for a range of values of a normally distributed random variable when given the mean and standard deviation.
Suppose ACT scores for a particular population are normally distributed with mean 21 and standard deviation 2. What proportion of the population has scores between 23 and 29?

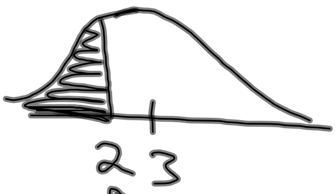
$P(23 < A < 29) = .1586$

$N(21, 2)$

μ σ
normalcdf(23, 29, 21, 2)

Normal dist.

Suppose a breed of chicken lays eggs with weights that are normally distributed with mean 3 ounces and standard deviation 0.5 ounces. What proportion of these eggs weigh less than 2 ounces?



$N(3, .5)$

$P(E < 2) = .025$

.0228

68-95-99.7 rule

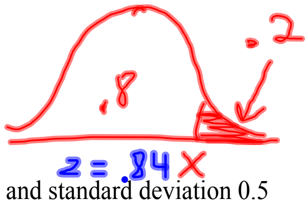
2(.5)

6. Calculate a value of a normally distributed random variable when given the mean, standard deviation, and a probability.
Suppose ACT scores for a particular population are normally distributed with mean 21 and standard deviation 2. How well does a student have to score to be in the top 20% of the population?

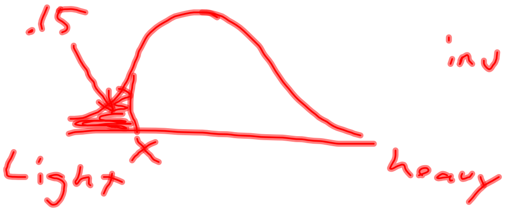
$N(21, 2)$

22.68 or better

$$.84 = \frac{x - 21}{2}$$



Suppose a breed of chicken lays eggs with weights that are normally distributed with mean 3 ounces and standard deviation 0.5 ounces. The lightest 15% of eggs weigh less than what number of ounces?



invNorm(.15, 3, .5) = 2.48 oz

μ σ

egg wts. ~Normal dist

7. Calculate the mean for a combination of independent random variables. Suppose a breed of chicken lays eggs with weights that are normally distributed with mean 3 ounces and standard deviation 0.5 ounces. Using these eggs, what is the mean weight of a 3 egg omelet?

$\mu_{E+E+E} = 3 + 3 + 3 = 9oz$

Suppose a game requires a 4-sided die and a 6-sided die to be rolled together. What is the mean of Y+Z as defined above?

$\mu_{Y+Z} = \mu_Y + \mu_Z$
$3.5 + 2.5 = 6$

Suppose a game requires three 4-sided dice and two 6-sided dice to be rolled together. What is the mean of $Y_1 + Y_2 + Z_1 + Z_2 + Z_3$ as defined above?

$\mu = 3.5 + 3.5 + 2.5 + 2.5 + 2.5 = 14.5$

Suppose a game requires that we take the outcome of rolling a 6-sided die and subtract the outcome of rolling a 4-sided die. What is the mean of Y-Z as defined above?

$\mu_{Y-Z} = \mu_Y - \mu_Z$
$= 3.5 - 2.5 = 1$

8. Calculate the variance and standard deviation for a combination of independent random variables.
Suppose a breed of chicken lays eggs with weights that are normally distributed with mean 3 ounces and standard deviation 0.5 ounces. Using these eggs, what is the standard deviation for weight of a 3 egg omelet?

$\sigma = .866$

Suppose a game requires a 4-sided die and a 6-sided die to be rolled together. What is the standard deviation of Y+Z as defined above?

$\sigma^2_{Y+Z} = \sigma^2_Y + \sigma^2_Z$

$(1.708)^2 + (1.118)^2 = 4.167$

$\sigma_{Y+Z} = \sqrt{4.167}$
$= 2.04$

Suppose a game requires three 4-sided dice and two 6-sided dice to be rolled together. What is the standard deviation of $Y_1 + Y_2 + Z_1 + Z_2 + Z_3$ as defined above?

$\sigma^2 = (1.708)^2 + (1.708)^2 + (1.118)^2 + (1.118)^2 + (1.118)^2$

$\sigma^2 = 9.584$ so $\sigma = \sqrt{9.584} = 3.1$

Suppose a game requires that we take the outcome of rolling a 6-sided die and subtract the outcome of rolling a 4-sided die. What is the standard deviation of Y-Z as defined above?

$\sigma^2_{Y-Z} = \sigma^2_Y + \sigma^2_Z$
$= (1.708)^2 + (1.118)^2 = 4.167$

$\sigma = \sqrt{4.167}$
$= 2.04$

9. Determine the effect of a linear transformation on the mean of a random variable.
Suppose the eggs described above are weighed in grams, rather than ounces. If 1 ounce = 28 grams, what is the mean weight of the eggs in grams?

$$\mu_E = 3\,\cancel{oz} \cdot \frac{28\,g}{1\,\cancel{oz}} = 84\,g$$

If I roll a 4-sided die and then double the result (2Z), what is the mean?

$$\mu_{2Z} = 2 \cdot \mu_Z = 2(2.5) = 5$$

If I roll a 6-sided die and then triple the result (3Y), what is the mean?

$$\mu_{3Y} = 3 \cdot \mu_Y = 3(3.5) = 10.5$$

$\sigma_E = .5 \quad \sigma_E^2 = .25$

10. Determine the effect of a linear transformation on the variance and standard deviation of a random variable.
Suppose the eggs described above are weighed in grams, rather than ounces. If 1 ounce = 28 grams, what are the variance and standard deviation of the weight of the eggs in grams?

$\sigma_E^2 = (.5)^2_{oz^2} \cdot (28)^2_{\frac{g^2}{oz^2}} = 196 g^2 \qquad \sigma = \sqrt{196 g^2} = 14 g$

If I roll a 4-sided die and then double the result (2Z), what are the variance and standard deviation?

$\sigma_{2Z}^2 = \sigma_Z^2 \cdot 2^2 = 1.25 \cdot 4 = 5 \qquad \sigma_{2Z} = \sqrt{5} = 2.24$

If I roll a 6-sided die and then triple the result (3Y), what are the variance and standard deviation?

$\sigma_{3Y}^2 = \sigma_Y^2 \cdot 3^2 = (1.708)^2 \cdot 9 = 26.253$

$\sigma_{3Y} = \sqrt{26.253} = 5.12$

11. A combination of several parts of these previous problems.
Suppose some eggs have a mean weight of 3 ounces and standard deviation of 0.5 ounces, some sausage links have a mean weight of 2 ounces and a standard deviation of 0.3 ounces, and some bread weighs an average of 1 ounce per slice with the standard deviation of 0.2 ounces. Assuming all these are normally distributed, then if a restaurant offers a breakfast platter of 3 eggs, 2 sausage links, and 2 slices of toast, what is the probability that the platter weighs more than 16 ounces?

$E \sim N(3, .5)$ $\quad$ $P = E + E + E + S + S + T + T$

$S \sim N(2, .3)$ $\quad$ $\mu_P = 3 + 3 + 3 + 2 + 2 + 1 + 1$

$T \sim N(1, .2)$ $\quad$ $= 15 \text{ oz}$

$$\sigma_P^2 = (.5)^2 + (.5)^2 + (.5)^2 + (.3)^2 + (.3)^2 + (.2)^2 + (.2)^2$$
$$= 1.01 \text{ oz}^2$$

$$\sigma_P = \sqrt{1.01} = 1.005 \text{ oz}$$

$$P \sim N(15, 1.005)$$

$$P(P > 16) = .1599$$