8.2 Geometric Distributions

Traits of the **geometric setting**:

1. Each observation has just 2 outcomes: a success and a failure.
2. The n observations are all independent.
3. The variable counts the number of trials needed to obtain the first success. (This was different in 8.1.)
4. The probability, p, of a success is the same for each observation.

Geometric mnemonic: BINS

Binary-- just 2 outcomes: success or failure

Independent trials (success probability doesn't change)
(close enough if we pick <10% of population)

Open-ended number of trials

Success probability is = for each trial before we begin

Ex. rock/paper/scissors

RR	RP	RS
PR	PP	PS
SR	SP	SS

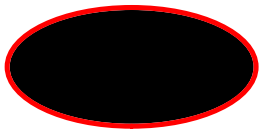
in one game,
someone will win or there's a tie

If X = # of games played until there's a winner, then X is a geometric random variable with the probability of a success p = 6/9 = 2/3.

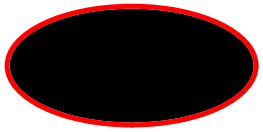
LET'S PLAY...

Normal, Binomial, or Geometric?

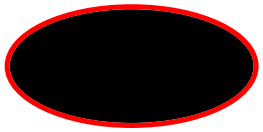
we record the # of defective bulbs in a box.



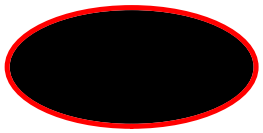
we record the # of hours the bulbs last.



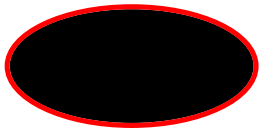
we record the # of bulbs checked until we find a non-defective bulb at the factory.



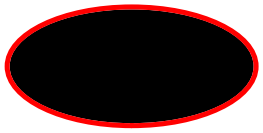
we record the weight of each bulb.



we record the number of bulbs that are not defective in a box.



we record how many boxes must be checked until we find one in which both bulbs are defective.



geometric probability

that the 1st success occurs on the nth trial is

$$P(X=n)=(1-p)^{n-1}p$$

ex. We have a limitless supply of light bulbs.
Suppose P(defective bulb) = 0.1.
Apply the formula above to find:

...the probability of a defective bulb on the first try.

...the probability that it takes 2 tries to find a defective bulb.

...the probability that it takes 4 tries to find a defective bulb.

2nd VARS
D:geometpdf
geometpdf(p,X)

ex. Apply the formula to find:

...the probability that a family has a girl right away.

...the probability that a family has a boy then a girl.

...the probability that a family has all boys
until having a girl as the 5th child.

...the probability that it takes 10 children
for a family to finally have a girl.

Ex. Suppose 20% of cars contain jumper cables.
Assuming independence, what's the probability I ask up to 3 people in order to find cables?

not independent if I ask others who...

- are also looking for jumper cables
- who all came in the same car
- who all belong to the International Anti-Jumper Cable Association

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If X is a geometric random variable with probability of success p on each trial, then the **mean,** or **expected value,** of X is:

$$\mu = 1/p$$

The mean tells you the expected number of trials required to get the first success. (The variance formula is not an AP topic.)

Ex. What's the expected # of light bulbs inspected (p=0.1) until I find a defective one?

Ex. If 20% of cars contain jumper cables, How many people you would expect to ask in order to find someone with cables?

Ex. For rock/paper/scissors, what's the average # of rounds in order for one of them to win?

The probability that it takes more than n trials to see the first success is $P(X>n)=(1-p)^n$

Ex. For light bulbs, assuming p=0.1, what's the probability we inspect more than 4 bulbs to find our first defective bulb?

What's the probability we inspect more than 12 bulbs to find a defective bulb?

For the number of girls in a family, what is the probability that it takes more than 3 children before a couple has a girl?

What is the probability that it takes more than 4 children before a couple has a girl?

Ex. If 20% of cars jumper cables, what's the probability that I have to ask more than 5 people to find cables?

For light bulbs, let p=0.1, the probability that I inspect up to 5 bulbs to find the first defective bulb: (the 1st or the 2nd or the 3rd or the 4th or the 5th), we add each of the geometric probabilities:
P(X=1)+P(X=2)+P(X=3)+P(X=4)+P(X=5)

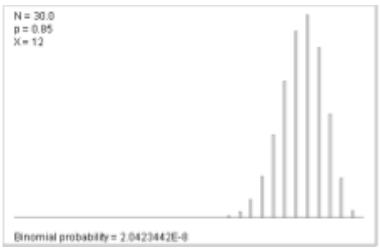
This is the **geometric cumulative distribution function** on the calculator.
2nd VARS E:geometcdf
geometcdf(p,X)

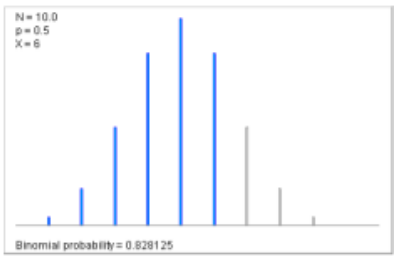
Ex. If 20% of cars have jumper cables, what's the probability I have to ask up to 4 people to find cables?

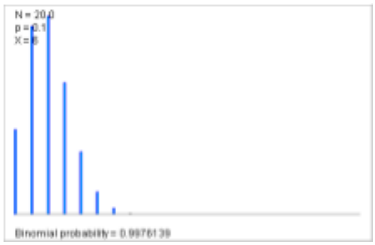
Ex. What's the probability it would take up to 5 tosses to get tails?

Ex. What's the probability that I'd have to roll a die up to 5 times in order to roll a 6?

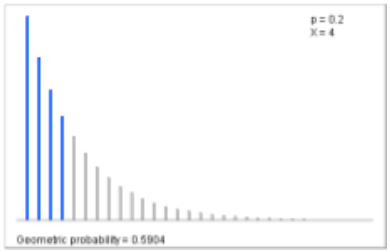
Binomial Distributions

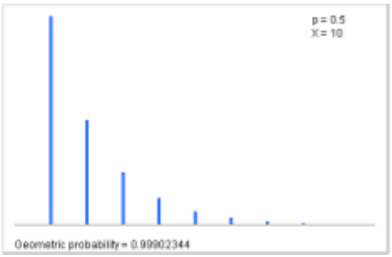


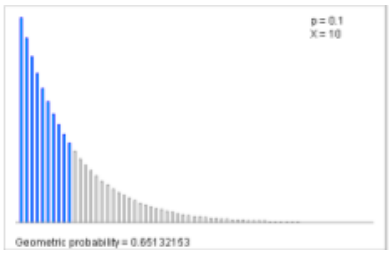




Geometric Distributions







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