8.2 Geometric Distributions

Traits of the **geometric setting**:

1. Each observation has just 2 outcomes: a success and a failure.
2. The n observations are all independent.
3. The variable counts the number of trials needed to obtain the first success. (This was different in 8.1.)
4. The probability, p, of a success is the same for each observation.

Geometric mnemonic: BIOS

Binary-- just 2 outcomes: success or failure
Independent trials (success probability doesn't change)
(close enough if we pick <10% of population)
Open-ended number of trials
Success probability is = for each trial before we begin

Ex. rock/paper/scissors

RR	RP	RS
PR	PP	PS
SR	SP	SS

in one game,
someone will win or there's a tie

If X = # of games played until there's a winner, then X is a geometric random variable with the probability of a success p = 6/9 = 2/3.

LET'S PLAY...

Normal, Binomial, or Geometric?

we record the # of defective bulbs in a box.	Binomial
we record the # of hours the bulbs last.	Normal
we record the # of bulbs checked until we find a non-defective bulb at the factory.	Geometric
we record the weight of each bulb.	Normal
we record the number of bulbs that are not defective in a box.	Binomial
we record how many boxes must be checked until we find one in which both bulbs are defective.	Geometric

<u>geometric probability</u>
that the 1st success occurs on the nth trial is

$$P(X=n)=(1-p)^{n-1}p$$

ex. We have a limitless supply of light bulbs.
Suppose P(defective bulb) = 0.1.
Apply the formula above to find:

...the probability of a defective bulb on the first try.

$(.1)^1$

...the probability that it takes 2 tries to find a defective bulb.

$(.9)^1(.1)^1 = .09$

...the probability that it takes 4 tries to find a defective bulb.

$(.9)(.9)(.9)(.1)$

2nd VARS
D:geometpdf
geometpdf(p,X)

$(.9)^3(.1)$

$.0729$

ex. Apply the formula to find:

...the probability that a family has a girl right away.

.5

...the probability that a family has a boy then a girl.

$(\underset{B}{.5})(\underset{G}{.5}) = .25$

...the probability that a family has all boys
until having a girl as the 5th child.

$\underline{(.5)(.5)(.5)(.5)(.5)} = .03125$ geometpdf(.5,5)

...the probability that it takes 10 children
for a family to finally have a girl.

$\text{geometpdf}(\overset{p}{.5},\overset{x}{10})$

$= .000977$

9 boys & 1 girl

$(.5)^9(.5)$

Ex. Suppose 20% of cars contain jumper cables.
Assuming independence, what's the probability I ask up to 3 people in order to find cables?

$$
\begin{aligned}
.2 &= .2 \\
(.8)(.2) &= .16 \\
(.8)(.8)(.2) &= + .128 \\
\hline
& \quad .488
\end{aligned}
$$

not independent if I ask others who...

- are also looking for jumper cables
- who all came in the same car
- who all belong to the International Anti-Jumper Cable Association

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If X is a geometric random variable with probability of success p on each trial, then the **mean,** or **expected value,** of X is:

$$\mu = 1/p$$

The mean tells you the expected number of trials required to get the first success. (The variance formula is not an AP topic.)

P(Heads) = .5

$\mu = \frac{1}{.5} = 2$

P(1 on a die) = $\frac{1}{6}$

$\mu = \frac{1}{\left(\frac{1}{6}\right)} = 6$

Ex. What's the expected # of light bulbs inspected (p=0.1) until I find a defective one?

$$\mu = \frac{1}{.1} = 10$$

Ex. If 20% of cars contain jumper cables, How many people you would expect to ask in order to find someone with cables?

$$\mu = \frac{1}{.2} = 5$$

Ex. For rock/paper/scissors, what's the average # of rounds in order for one of them to win?

$$\mu = \frac{1}{\left(\frac{2}{3}\right)} = \frac{3}{2} = 1.5$$

The probability that it takes <u>more</u> than n trials to see the first success is $P(X>n)=(1-p)^n$

P(no defect) = 1 - p = .9

Ex. For light bulbs, assuming p=0.1, what's the probability we inspect more than 4 bulbs to find our first defective bulb?

$(.9)(.9)(.9)(.9) = (.9)^4 = .6561$

What's the probability we inspect more than 12 bulbs to find a defective bulb?

$(.9)^{12} = .282$

For the number of girls in a family, what is the probability that it takes more than 3 children before a couple has a girl?

$\overset{B}{(.5)}\overset{B}{(.5)}\overset{B}{(.5)} = .125$

What is the probability that it takes more than 4 children before a couple has a girl?

$(.5)^4 = .0625$

$(.5)(.5)(.5)(.5)$

Ex. If 20% of cars jumper cables, what's the probability that I have to ask more than 5 people to find cables?

$P(\text{cables}) = .2$

$P(\text{cables}^c) = .8$

$(.8)(.8)(.8)(.8)(.8) = (.8)^5 = .327$

For light bulbs, let p=0.1, the probability that I inspect <u>up to</u> 5 bulbs to find the first defective bulb: (the 1st or the 2nd or the 3rd or the 4th or the 5th), we <u>add</u> each of the geometric probabilities:
P(X=1)+P(X=2)+P(X=3)+P(X=4)+P(X=5)

$.1 + (.9)(.1) + \cdots + (.9)^4(.1)$

$.1 + (.9)(.1) + (.9)^2(.1) + (.9)^3(.1) + (.9)^4(.1)$

This is the **<u>geometric cumulative distribution function</u>** on the calculator.
2nd VARS E:geometcdf
geometcdf(p,X)

geometcdf(.1, 5)
= .41

Ex. If 20% of cars have jumper cables, what's the probability I have to ask up to 4 people to find cables?

$.2 + (.8)(.2) + \cdots + (.8)^3(.2) = .59$

geometcdf(.2, 4)

X = 4
p = .2

There's a .59 chance of getting cables by the time I ask 4 people

Ex. What's the probability it would take up to 5 tosses to get tails?

$.5 + (.5)(.5) + \cdots + (.5)^4(.5)$

$x = 5$

$p = .5$

$= .969$

I have a .969 chance of having tails by the 5th toss.

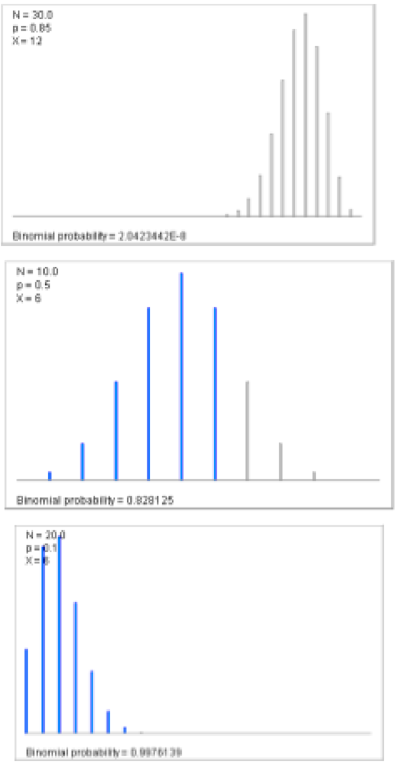
geometcdf(.5, 5)

Ex. What's the probability that I'd have to roll a die up to 5 times in order to roll a 6?

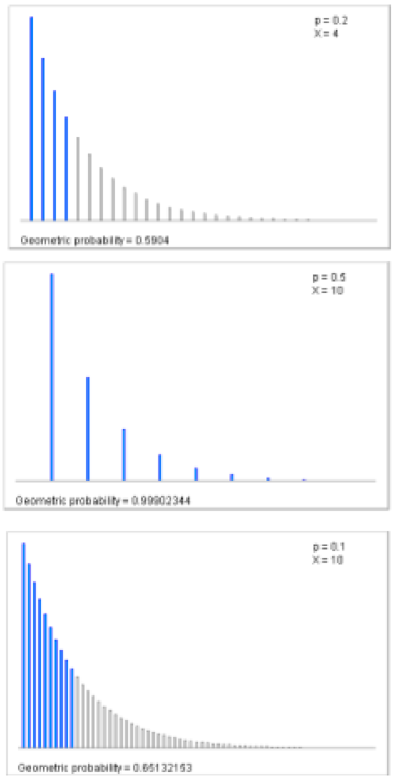
$\frac{1}{6} + \left(\frac{5}{6}\right)\left(\frac{1}{6}\right) + \cdots + \left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right) = .598$

geometcdf(1/6, 5)

Binomial Distributions



Geometric Distributions



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