# 9.2 Sample Proportions

sample proportion has a distribution
with a mean and standard deviation

http://www.rossmanchance.com/applets/GettysburgSample/GettysburgSample.html

Draw 100 samples of size 10. Check each mean and standard deviation in the Long and Noun graphs.

http://www.rossmanchance.com/applets/senators/samplesenators.html

Draw 100 samples of size 10. Check each mean and standard deviation in the Republican and Female graphs.

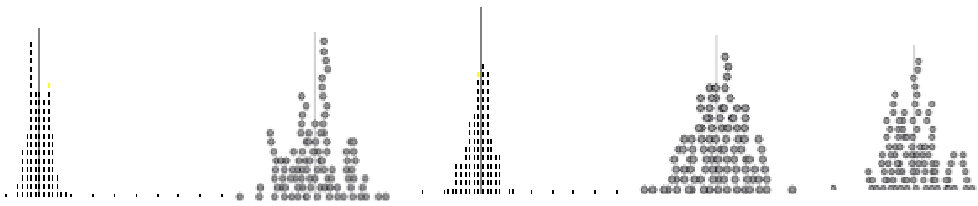
large university lecture hall courses
% who have had dental fillings

Same % in every auditorium?

No.  We usually expect different samples to yield different statistics.

% who have fillings in all groups of 300 students
graph each % found

probably a bell curve, maybe almost Normal, like these:



sampling distribution of the sample proportion:
S approx. Normal
O few, if any
C ≈ % who have fillings in population
S since n=300, spread's smaller than if n had 100 & larger than if n had been 400

Sampling Distribution of the Sample Proportion is the distribution of sample proportions from all possible samples of this size.

population → population proportion p

↓

SRS of size n → sample proportion $\hat{p}$ ← These have a distribution.

**mean** of the sampling distribution of $\hat{p}$ is exactly p.

$$\mu_{\hat{p}} = p$$

**standard deviation** of the sampling distribution of $\hat{p}$ is

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

as long as the population is at least 10 times as large as the sample ($N \geq 10n$).

Why N>10n? Because of the methods we use. Larger samples are so much better that our standard deviation formula is no longer a good estimate. If our sample is more than 1/10 the population, the assumptions, conditions, methods, and formulas we have won't work.

The Success/Failure Condition for Normal Approx.
If $np \geq 10$ & $n(1-p) \geq 10$, then the sampling distribution of $\hat{p}$ is approximately Normal.

Central Limit Theorem

Draw an SRS of size n from **any population whatsoever** with proportion p. When n is large enough, the sampling distribution of sample proportion $\hat{p}$ is close to the normal distribution $N(p, \sqrt{\frac{p(1-p)}{n}})$.

$np \geq 10$

and

$n(1-p) \geq 10$

Ch. 9
← %

$$N\left(np, \sqrt{\left(np\left(1-p\right)\right)}\right)$$

Ch. 8
← counts

Ex. Consider 100 tosses of a coin.

- The mean proportion of tails =

- Verify n for the standard deviation:

The standard deviation for the proportion of tails =

- Verify Success/Failure Condition for Normal Approximation:

The sampling distribution of the percent of tails is approximately Normal:

So, by the 68-95-99.7 Rule,

- There's a 68% chance that the proportion of tails will be between 0.45 and 0.55 for 100 tosses.
- There's a 95% chance that the proportion of tails will be between 0.40 and 0.60 for 100 tosses.
- There's a 99.7% chance that the proportion of tails will be between 0.35 and 0.65 for 100 tosses.

It would be unusual to toss a coin 100 times and have under 35% tails or over 65% tails.

Ex. A psychologist studies the existence of ESP by having a participant guess which of 6 cards the psychologist has selected. This is repeated 90 times per participant with many people.
For guessing, P(correct guess)=1/6.

- The mean proportion of correct guesses=

- Verify n for the standard deviation:

The standard deviation for the proportion of correct guesses =

- Verify Success/Failure Condition for Normal Approximation:

The sampling distribution of the percent of correct guesses is approximately Normal:

So, by the 68-95-99.7 Rule,
There's a 99.7% chance that participant will guess between _____% & _____% of the 90 guesses correctly.

Almost everyone would guess _____% to _____% correctly.

If a participant guesses more than _____% correctly, that is unusually high (fewer than _____% correct would be unusually low).