9.3 Sample Means

sample mean has a distribution
with a mean and standard deviation

http://www.rossmanchance.com/applets/SampleData/SampleData.html

Draw 100 samples of size 10. Check each mean and standard deviation in the Year graph.

http://www.ruf.rice.edu/~lane/stat_sim/sampling_dist/

Draw 100 samples of size 5, then increase the size each time. Check what happens to the mean and standard deviation as n increases.

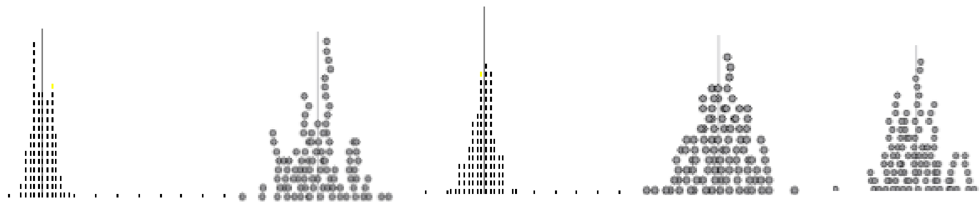
large university lecture hall courses
mean number of songs on their iPods

Same mean in every auditorium?

No. We usually expect different samples to yield different statistics.

mean number of songs on their iPods in all groups of 300 students
graph each mean found

probably a bell curve, maybe almost Normal, like these:



sampling distribution of the sample mean:
S approx. Normal
O few, if any
C $\approx$ mean number of songs on iPods in population
S since n=300, spread's smaller than if n had 100 & larger than if n had been 400

Sampling Distribution of the Sample Mean is the distribution of sample means from all possible samples of this size.

population → population mean μ

↓

SRS of size n → sample mean $\bar{x}$ ← These have a distribution.

mean of the sampling distribution of $\overline{x}$ is exactly μ.

$$\mu_{\overline{x}} = \mu$$

standard deviation of the sampling distribution of $\hat{p}$ is

$$\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}}$$

as long as the population is at least 10 times as large as the sample ($N \geq 10n$).

Why N>10n? Because of the methods we use. Larger samples are so much better that our standard deviation formula is no longer a good estimate. If our sample is more than 1/10 the population, the assumptions, conditions, methods, and formulas we have won't work.

Normal population $\Rightarrow$ Normal sampling distribution of the sample mean

non-Normal population $\Rightarrow$ nearly Normal sampling distribution of the sample mean **<u>if</u>** the sample size is at least 30 ($n \geq 30$)

Central Limit Theorem

Draw an SRS of size n from **any population whatsoever** with mean μ. When n is large enough, the sampling distribution of sample mean $\overline{x}$ is close to the normal distribution $N(\mu, \frac{\sigma}{\sqrt{n}})$.

$n \geq 30$

Ex. Consider adults of some snake species that is N(40,6) in cm.
Take random samples of n = 4 adult snakes.

- The mean of the sample means =

- Verify n for the standard deviation

The standard deviation for the mean length =

- Verify condition for a Normal approximation:

The sampling distribution of the sample mean is approximately Normal:

Then the 68-95-99.7 Rule says that

- 68% of samples of n=4 such adult snakes will have an average length between 37 and 43 cm.
- 95% of samples of n=4 such adult snakes will have an average length between 34 and 46 cm.
- 99% of samples of n=4 such adult snakes will have an average length between 31 and 49 cm.

It would be rare for a sample of 4 such adult snakes to have an average length over 49 cm or under 31 cm.

Ex. One year, the mean ACT score was 20.8 & the standard deviation was 4.8. Suppose we take a random samples of n = 36 scores.

- The mean sample mean ACT score=

- Verify n for the standard deviation:

The standard deviation for the sample mean ACT score=

Verify the Condition for Normal Approximation:

The sampling distribution of the sample mean ACT score
is approximately Normal:

So, by the 68-95-99.7 Rule,
There's a 99.7% chance that samples of n=36 ACT scores will have an average between _____ & _____.

Almost every sample of n=36 ACT scores will have a mean between _____ & _____.

If a sample of n=36 ACT scores has a mean more than _____, that is unusually high (lower than _____ would be unusually low).