Name ______________________________

Chapter 10 Learning Objectives	**Section**	**Related Example on Page(s)**	**Relevant Chapter Review Exercise(s)**	**Can I do this?**
Describe the shape, center, and spread of the sampling distribution of $\hat{p}_1 - \hat{p}_2$.	10.1	615	R10.2	
Determine whether the conditions are met for doing inference about $p_1 - p_2$.	10.1	617	R10.5, R10.6	
Construct and interpret a confidence interval to compare two proportions.	10.1	617	R10.2	
Perform a significance test to compare two proportions.	10.1	622, 625	R10.5	
Describe the shape, center, and spread of the sampling distribution of $\bar{x}_1 - \bar{x}_2$.	10.2	638	R10.3	
Determine whether the conditions are met for doing inference for $\mu_1 - \mu_2$.	10.2	641	R10.3, R10.4, R10.6	
Construct and interpret a confidence interval to compare two means.	10.2	641	R10.4	
Perform a significance test to compare two means.	10.2	645	R10.7	
Determine when it is appropriate to use two-sample *t* procedures versus paired *t* procedures.	10.2	650	R10.1, R10.7	

10.1 Comparing Two Proportions

Is it harder to sink a golf ball on a putt with distractions? To investigate, a golf team member went to a golf course and attempted 30 putts on the same hole from the same location. Fifteen of the putts were attempted without any distractions and the other 15 were attempted with her friends trying everything they could to distract her. The order of the 30 shots was determined at random.

Why was it important that the order of the putts was determined at random, rather than doing all of one type of putt before the other type of putt?

The player made 8/15 (53%) of her putts in the distraction-free environment and only 4/15 (27%) of her putts in the environment with distractions, for a difference of 53% – 27% = 26%. Identify two plausible explanations for why the golfer performed better in the distraction-free environment.

Let’s conduct a simulation to determine if there is convincing evidence that the golfer is better in a distraction-free environment. If we let p be the proportion of successful putts under some condition, then

$H_0: p_{distraction} = p_{silent}$

$H_a: p_{distraction} < p_{silent}$

Read 612–615

The **sampling distribution of the difference between two proportions** is the dist. of all possible differences $\hat{p}_1 - \hat{p}_2$ for all samples of a specified size.

What are the shape, center, and spread of the sampling distribution of $\hat{p}_1 - \hat{p}_2$? Are there any conditions that need to be met?

Choose an SRS of size n_1 from Population 1 with proportion of successes p_1 and an independent SRS of size n_2 from Population 2 with proportion of successes p_2.

- **Shape:** When n_1p_1, $n_1(1 - p_1)$, n_2p_2, and $n_2(1 - p_2)$ are all at least 10, the sampling distribution of $\hat{p}_1 - \hat{p}_2$ is approximately Normal.
- **Center:** The mean of the sampling distribution of $\hat{p}_1 - \hat{p}_2$ is $p_1 - p_2$.
- **Spread:** The standard deviation of the sampling distribution of $\hat{p}_1 - \hat{p}_2$ is

$$\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

as long as each sample is no more than 10% of its population.

Nick goes to North and Chris attends Central, both large schools. Suppose that at North, 70% of the smartphones are iPhones and that at Central, 60% of the phones are iPhones. Nick selects a random sample of 100 smartphone owners in his school and Chris selects a random sample of 80 smartphone owners at his school. Let $\hat{p}_N - \hat{p}_C$ be the difference in the sample proportion of smartphones that are iPhones at these schools.

(a) What is the shape of the sampling distribution of $\hat{p}_N - \hat{p}_C$? Why?

(b) Find the mean of the sampling distribution of $\hat{p}_N - \hat{p}_C$. Show your work.

(c) Find the standard deviation of the sampling distribution of $\hat{p}_N - \hat{p}_C$. Show your work.

HW #23: page 629 (1, 3, 29, 30)

10.1 Significance Tests for a Difference in Proportions

Read 619–624

What are the conditions for conducting a two-sample *z* test for a difference in proportions?

- **Random:** The data come from two independent random samples or from two groups in a randomized experiment.
  - **10%:** When sampling without replacement, check that $n_1 \le \frac{1}{10}N_1$ and $n_2 \le \frac{1}{10}N_2$.
- **Large Counts:** The counts of "successes" and "failures" in each sample or group—$n_1\hat{p}_1$, $n_1(1-\hat{p}_1)$, $n_2\hat{p}_2$, $n_2(1-\hat{p}_2)$—are all at least 10.

Notice that these are just double the conditions for a one-sample z test for p; we do each one twice.

The pooled (or combined) sample proportion is just the overall proportion of successes:

$$\hat{p}_C = \frac{\text{count of successes in both samples combined}}{\text{count of individuals in both samples combined}} = \frac{X_1 + X_2}{n_1 + n_2}$$

We pool the sample proportions when we conduct a hypothesis test on 2 proportions because the H_0 says we are assuming that two proportions are equal.

This is the test statistic for a two-sample z test for a difference in proportions:
The formula sheet has this in words.

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - 0}{\sqrt{\frac{\hat{p}_C(1-\hat{p}_C)}{n_1} + \frac{\hat{p}_C(1-\hat{p}_C)}{n_2}}}$$

The test statistic measures the number of standard deviations between our statistic ($\hat{p}_1 - \hat{p}_2$) and our hypothesized parameter ($p_1 - p_2$).

A national health survey found that, of 3200 randomly selected teenagers in 1988–1994, 18.5% showed slight to moderate hearing loss. In a similar study of 1800 teenagers in 2005–2006, 24.8% showed slight to moderate hearing loss. (slight loss is >15 dB, mild to moderate is >25 dB)

(a) Do these data give convincing evidence that the proportion of all teens with hearing loss has increased?

(b) Between the two studies, in 1991, Apple introduced the iPod. If the results of the test are statistically significant, can we blame iPods for the increased hearing loss in teenagers?

It is OK to use your calculator for the Do step, just be sure to show your calculation of the successes and failures in the "plan" step.

HW #24: page 630 (5, 7, 13, 15, 19)
CIs for the Difference of Two Proportions, Inference for Experiments
Read 616–619
What are the conditions for calculating a two-sample *z* interval for $p_1 - p_2$?

- **Random:** The data come from two independent random samples or from two groups in a randomized experiment.
  - **10%:** When sampling without replacement, check that $n_1 \leq \frac{1}{10}N_1$ and $n_2 \leq \frac{1}{10}N_2$.
- **Large Counts:** The counts of "successes" and "failures" in each sample or group—$n_1\hat{p}_1$, $n_1(1-\hat{p}_1)$, $n_2\hat{p}_2$, $n_2(1-\hat{p}_2)$—are all at least 10.

What is the standard error of $\hat{p}_1 - \hat{p}_2$?

$$SE_{\hat{p}_1-\hat{p}_2} = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

How is this different than the standard deviation of $\hat{p}_1 - \hat{p}_2$ on page 2 of these notes?

Why is this different than the standard error we used for significance tests on page 3 of these notes?

What is the formula for a two-sample *z* interval for $p_1 - p_2$? Is this on the formula sheet?

When the conditions are met, an approximate C% confidence interval for $\hat{p}_1 - \hat{p}_2$ is

$$(\hat{p}_1 - \hat{p}_2) \pm z^*\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

where z^* is the critical value for the standard Normal curve with C% of its area between $-z^*$ and z^*.

Have opinions changed about the death penalty? Gallup regularly asks random samples of U.S. adults their opinion on a variety of issues. In a poll of 1017 U.S. adults in October 2014, 63% responded that they “were in favor of the death penalty for those convicted of murder.” In a similar poll in September 1994, 80% indicated that they favored the death penalty in such situations.

(a) Explain why we should use a confidence interval to estimate the change in opinion rather than just saying that the percentage decreased by 17 percentage points.

(b) Assuming that both polls used the same sample size, use the results of these polls to construct and interpret a 90% confidence interval for the change in the proportion of U.S. adults who would say they favor the death penalty for convicted murderers.

(c) Based on the interval, is there convincing evidence that opinions about the death penalty have changed?
Read 625–627

Common mistakes when defining parameters in experiments:	Correct way to do this instead:
Referring to samples	Referring to populations
Referring to statistics like $\bar{x}$ and $\hat{p}$	Referring to parameters like μ and p
Saying something in past tense like "all people who took drug A…"	Saying something in conditional (or hypothetical or future) tense like "all people who would take (will take) drug A…"

In a 2002 study reported in the Archives of Pediatric and Adolescent Medicine, 26 patients with warts were randomly assigned to apply duct tape to their warts for 2 months and 25 patients with warts were treated with liquid Nitrogen cryotherapy for 10 seconds every 2-3 weeks for up to 6 treatments. At the end of the study, 22 of the duct tape group and 15 of the cryotherapy group had complete resolution of their warts. What condition for inference is not met by this study?

Suppose the results were 66/78 using duct tape had complete resolution and 45/75 using cryotherapy had resolution. Do the results based on these revised numbers give convincing evidence that duct tape is better than cryotherapy at treating warts?

HW #25: page 630 (9, 11, 21, 23)

Significance Tests for the Difference of Two Means

Read 634–639

The **<u>sampling distribution of the difference between two means</u>** is the distribution of all possible differences $\bar{x}_1 - \bar{x}_2$ for all samples of a specified size.

What are the shape, center, and spread of the sampling distribution of $\bar{x}_1 - \bar{x}_2$? Are there any conditions that need to be met?

THE SAMPLING DISTRIBUTION OF $\bar{x}_1 - \bar{x}_2$

Choose an SRS of size n_1 from Population 1 with mean μ_1 and standard deviation σ_1 and an independent SRS of size n_2 from Population 2 with mean μ_2 and standard deviation σ_2.

- **Shape:** When the population distributions are Normal, the sampling distribution of $\bar{x}_1 - \bar{x}_2$ is Normal. In other cases, the sampling distribution of $\bar{x}_1 - \bar{x}_2$ will be approximately Normal if the sample sizes are large enough ($n_1 \geq 30$ and $n_2 \geq 30$).
- **Center:** The mean of the sampling distribution of $\bar{x}_1 - \bar{x}_2$ is $\mu_1 - \mu_2$.
- **Spread:** The standard deviation of the sampling distribution of $\bar{x}_1 - \bar{x}_2$ is

$$\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

as long as each sample is no more than 10% of its population.

Read 639–640

This is the standard error of $\bar{x}_1 - \bar{x}_2$. $SE_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$

The formula sheet uses σ instead of s, so the formula sheet has the standard deviation and when we use s instead, we have the standard error. To interpret the standard error, we say that this is an estimate of the typical distance (or difference) between differences $\bar{x}_1 - \bar{x}_2$ and the difference $\mu_1 - \mu_2$.

This is the formula for the two-sample *t* statistic: $t = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$

It appears on the formula sheet in words and it measures the number of standard errors (which is an estimate of the standard deviation) between the observed difference of the sample means and the hypothesized difference of the population means.

These are the conditions for performing inference about $\mu_1 - \mu_2$:

CONDITIONS FOR PERFORMING INFERENCE ABOUT $\mu_1 - \mu_2$

- **Random:** The data come from two independent random samples or from two groups in a randomized experiment.
  - **10%:** When sampling without replacement, check that $n_1 \le \frac{1}{10}N_1$ and $n_2 \le \frac{1}{10}N_2$.
- **Normal/Large Sample:** Both population distributions (or the true distributions of responses to the two treatments) are Normal or both sample sizes are large ($n_1 \ge 30$ and $n_2 \ge 30$). If either population (treatment) distribution has unknown shape and the corresponding sample size is less than 30, use a graph of the sample data to assess the Normality of the population (treatment) distribution. Do not use two-sample *t* procedures if the graph shows strong skewness or outliers.

The two-sample *t* statistic has *close to* a t distribution—close enough for our purposes, as long as the conditions above are all met. We use a *t* statistic rather than a *z* statistic because we don't know the (population standard deviation) values of σ and are using (sample standard deviation) s values instead. Let the calculator calculate the degrees of freedom. *Please use the calculator for the do step!*

Read 644–649
A researcher filled (to recommended pressure in pounds/square inch (psi)) 25 randomly selected car tires with air (roughly 78% nitrogen) and 27 identically sized, randomly selected tires with compressed nitrogen (95% pure nitrogen). After a period of storage, the average decrease in pressure of the air-filled tires was 3.5 psi with a standard deviation of 0.409 psi. The average decrease of the nitrogen-filled tires was 2.2 psi with a standard deviation of 0.413 psi. Assume the conditions for inference are met.
(a) Do these data provide convincing evidence that nitrogen-filled tires maintain pressure better than air-filled tires?

Recall that the **p-value** is defined informally as the probability of obtaining a result equal to or "more extreme" than what was actually observed, when the null hypothesis is true.
(b) Interpret the *P*-value you got in part (a) in the context of this study.

It is OK to use your calculator for the "Do" step, but be sure to write down and identify (label) numbers you enter and always say "NO" to pooling when doing inference for means.

HW #26 page 632 (25-28), page 654 (31, 33, 45, 51)
Confidence Intervals for the Difference of Two Means
Read 641–643
This is the formula for the two-sample t interval for $\mu_1 - \mu_2$:

This formula on the formula sheet in words.

The conditions for this interval are the same as for the one-sample t interval, but we do each one twice.

TWO-SAMPLE *t* INTERVAL FOR A DIFFERENCE BETWEEN TWO MEANS

When the conditions are met, an approximate C% confidence interval for $\mu_1 - \mu_2$ is

$$(\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

Here, t^* is the critical value with C% of its area between $-t^*$ and t^* for the t distribution with degrees of freedom using either Option 1 (technology) or Option 2 (the smaller of $n_1 - 1$ and $n_2 - 1$).

Tim thinks one brand of sandwich cookies has more "stuff" in them, on average, than another brand. His friend Toni is not convinced. They decide to gather some data to see if there's a difference in the amount of "stuff" in the two brands. To investigate, they randomly selected a 30 cookies of each brand and carefully scraped off and weighed the "stuff". Here are their results (weighed in grams):

Brand A	Brand B
Sample size=30	Sample size=30
Sample mean=3.497	Sample mean=3.572
Sample standard deviation=0.125	Sample standard deviation=0.057

(a) Construct and interpret a 99% confidence interval for the difference in the mean weight of "stuff" in the two brands.

(b) Does your interval provide convincing evidence that there is a difference in the mean weight of "stuff" in the two brands?

HW #27: page 654 (35, 37, 43, 47, 49)

10.2 Using *t* Procedures Wisely / PROJECTS

Read 650–651

Which t procedure should be used?		
I have 1 set of data from 1 sample.	I have 2 sets of data that are **paired**, like from before-after studies or other matched pairs design.	I have 2 sets of data from… * 2 independent samples from 2 populations or * random assignment to 2 experimental treatments.
Use t procedures	Use t procedures on the **differences** of each pair	Use 2 sample t procedures
2:T-Test or 8:TInterval	2:T-Test or 8:TInterval	4:2-SampTTest or 0:2-SampTInt

Suppose you are designing an experiment to determine if people perform better on a logic puzzle when there are no distractions, such as intermittent loud noises. You have access to two classrooms and 30 volunteers who are willing to participate in your experiment. Oh, you also have a sound system with a recording of sounds from a large construction site.

(a) Design an experiment so that a two-sample *t* test would be the appropriate inference method.

(b) Design an experiment so that a paired *t* test would be the appropriate inference method.

(c) Which experimental design is better? Explain.

When doing two-sample *t* procedures, **<u>don't "pool"</u>** since we don't know that $\sigma_1 = \sigma_2$.

When doing two-sample test for a difference in proportions, we pooled because $H_0: p_1 = p_2$.

HW #28: page 659 (53–62)
Review Chapter 10
Frappy: 2011 #4 (Cholesterol drugs)
HW #29: page 664 Chapter 10 Review Exercises
Review Chapter 10
HW #30: page 666 Chapter 10 AP Practice Test
Chapter 10 Test