Name ______________________________

Chapter 8 Learning Objectives	**Section**	**Related Example on Page(s)**	**Relevant Chapter Review Exercise(s)**	**Can I do this?**
Determine the point estimate and margin of error from a confidence interval.	8.1	481	R8.2	
Interpret a confidence interval in context.	8.1	481, 484	R8.3, R8.4, R8.6, R8.7	
Interpret a confidence level in context.	8.1	484	R8.2	
Describe how the sample size and confidence level affect the length of a confidence interval.	8.1	Discussion on 487	R8.9	
Explain how practical issues like nonresponse, undercoverage, and response bias can affect the interpretation of a confidence interval.	8.1	Discussion on 488	R8.6	
State and check the Random, 10%, and Large Counts conditions for constructing a confidence interval for a population proportion.	8.2	494	R8.3	
Determine critical values for calculating a *C%* confidence interval for a population proportion using a table or technology.	8.2	497	R8.1	
Construct and interpret a confidence interval for a population proportion.	8.2	498, 500	R8.3, R8.6	
Determine the sample size required to obtain a *C%* confidence interval for a population proportion with a specified margin of error.	8.2	502	R8.5	
State and check the Random, 10%, and Normal/Large Sample conditions for constructing a confidence interval for a population mean.	8.3	516	R8.4	
Explain how the *t* distributions are different from the standard Normal distribution and why it is necessary to use a *t* distribution when calculating a confidence interval for a population mean.	8.3	Discussion on 511–512	R8.10	
Determine critical values for calculating a *C%* confidence interval for a population mean using a table or technology.	8.3	513	R8.1	
Construct and interpret a confidence interval for a population mean.	8.3	519	R8.4, R8.7	
Determine the sample size required to obtain a *C%* confidence interval for a population mean with a specified margin of error.	8.3	524	R8.8	

8.1 Confidence Intervals: The Basics

	Chapter 7	Chapter 8
We're told about:	The population's distribution	One sample
We try to figure out:	How statistics are distributed for repeated samples	The population mean or the population proportion

Read 477–480

What's a point estimate?

DEFINITION: Point estimator and point estimate

A **point estimator** is a statistic that provides an estimate of a population parameter. The value of that statistic from a sample is called a **point estimate.**

It's called a point estimate because, if we graph that single number on a number line, we plot a single point.

It's very unlikely that our statistic is exactly equal to the population parameter we're trying to estimate. However, if we use an <u>interval estimate</u> and say that we believe the population parameter is between ____ and ____, then we're more likely to be correct.

Here's the logic of confidence intervals.

1. Distance from $\bar{x}$ to μ is the same as the distance from μ to $\bar{x}$.
2. For 95% of samples, $\bar{x}$ will be within about 2 SD of μ.
3. Therefore, for 95% of samples, μ will be within about 2 SD of $\bar{x}$.

What is a confidence interval? What is the margin of error? Why do we include the margin of error?

DEFINITION: Confidence interval, margin of error, confidence level

A ***C*% confidence interval** gives an interval of plausible values for a parameter. The interval is calculated from the data and has the form

point estimate ± margin of error

The difference between the point estimate and the true parameter value will be less than the **margin of error** in *C*% of all samples.

The **confidence level *C*** gives the overall success rate of the method for calculating the confidence interval. That is, in *C*% of all possible samples, the method would yield an interval that captures the true parameter value.

Read 480–485

To interpret a confidence <u>interval</u>: *"We're 95% confident that the interval from __ to __ contains the true __."*

According to a Siena college survey published on Sept. 24, 2015, a 98% confidence interval for the true proportion of New York adults who support a $15 minimum wage is 59% ± 4%. This estimate was based on a random sample of 817 New York adults. Interpret this interval in context.

How do you interpret a confidence <u>level</u>? In other words, what does it mean to be 95% confident?

"If we were to take many, many samples and calculate many, many intervals, about 95% of the intervals will capture the true __ for the population."

To prepare for an upcoming event at our school's stadium, some parents attended a similar event at Central HS recently. To estimate how much money is spent, on average, at the concession stand at this event, the parents somehow selected a SRS of 50 individuals in the bleachers and asked them how much they spent on concessions. After collecting the data, they calculated a 95% confidence interval of $6.68 to $15.84.

(a) Interpret the confidence level.

(b) Interpret the confidence interval.

(c) What is the point estimate that was used to create the interval? What is the margin of error?

(d) A parent from Central mentioned to one of our parents that people at the event typically buy $17 on concessions at their event. Is there convincing evidence that the people attending Central's event do not spend that much, on average? Explain.

Read 485–488

Here is the formula for calculating a confidence interval, just as it appears on the formula sheet (in words).

CALCULATING A CONFIDENCE INTERVAL

The confidence interval for estimating a population parameter has the form

statistic ± (critical value) · (standard deviation of statistic)

where the statistic we use is the point estimator for the parameter.

To reduce the margin of error in a confidence interval, we can increase the sample size or decrease the confidence level.

A small margin of error is desirable because it makes our estimate more precise.

Drawbacks to these actions:

If we increase the sample size,

If we decrease the confidence level,

Two important things to remember when constructing and interpreting confidence intervals:
1. Our method assumes that the data come from a SRS of size n from the population of interest.
2. The margin of error covers only chance variation due to random selection (sampling) or random assignment (experiments).

In 2014 and 2015 surveys, researchers asked random samples of US teens and adults if they use Twitter. Overall, 33% of the teens said yes and 19% of the adults said yes. A 90% confidence interval for the true difference in the proportion of teens and adults who would say yes is 0.112 to 0.168.

(a) Interpret the confidence level.

(b) Interpret the confidence interval.

(c) Based on the interval, is there convincing evidence that the proportion of teens who would say yes is higher than the proportion of adults who would say yes? Explain.

(d) How would the interval be affected if we used a 99% confidence level instead of a 90% confidence level?

HW #6: page 489 (1, 3, 9–19 odd, 25, 26)

8.2 Confidence Intervals for a Proportion

Read 492–496

CONDITIONS FOR CONSTRUCTING A CONFIDENCE INTERVAL ABOUT A PROPORTION

- **Random:** The data come from a well-designed random sample or randomized experiment.
  - **10%:** When sampling without replacement, check that $n \leq \frac{1}{10}N$.
- **Large Counts:** Both $n\hat{p}$ and $n(1-\hat{p})$ are at least 10.

What happens if conditions are violated?

Random : p-hat might not be close to p.

10% : the standard deviation we calculate might be incorrect.

Large Counts : normal approximations might be incorrect.

Read 496–499

These are the formulas for: the <u>standard deviation</u> of the sample proportion and the <u>standard error</u> of the sample proportion

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} \qquad \qquad SE_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Notice the difference: The standard deviation uses p. The standard error uses p-hat.

How do you interpret each one?

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

The <u>standard deviation of the sample proportion</u> is the typical distance of the sample proportion, p-hat, from the population proportion, p, in repeated SRSs of size n.

$$SE_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

The <u>standard error of the sample proportion</u> is *an estimate of* the typical distance of the sample proportion, p-hat, from the population proportion, p, in repeated SRSs of size n.

The <u>standard deviation</u> formula <u>**IS**</u> on the formula sheet.

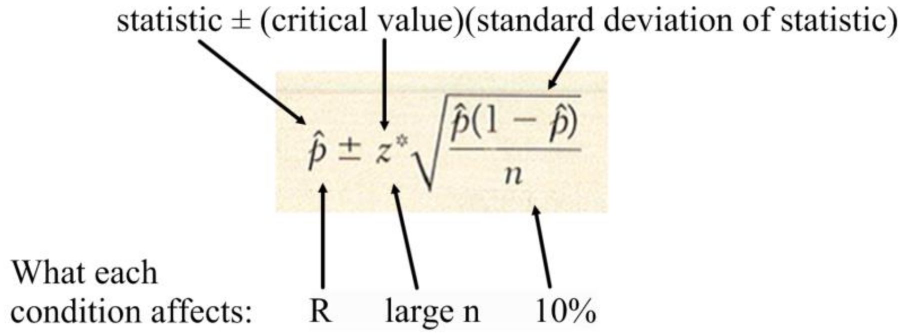
A <u>**critical value**</u> is a multiplier (a factor) that makes the interval wide enough to have the stated capture rate.

A z critical value (denoted z*) comes from using the Standard Normal distribution table or invNorm on the calculator. The * on z* reminds us that the z value came from the Standard Normal distribution, N(0,1), rather than from data.

Find the critical value z* for a 96% confidence interval for a proportion.

Find the critical value z* for a 90% confidence interval for a proportion.

The formula sheet has the formula for a confidence interval in words:



Students in an AP Statistics class want to estimate the proportion of pennies in circulation that are less than 10 years old. To do this, they gathered all the pennies they had in their pockets and purses. Overall, 45 of the 102 pennies they have are less than 10 years old.

(a) Identify the population and the parameter of interest.

(b) Check the conditions for calculating a confidence interval for the parameter.

(c) Construct a 99% confidence interval for the parameter.

(d) Interpret the interval in context.

(e) Is it plausible that more than 60% of all pennies in circulation are less than 10 years old?

HW #7: page 490 (10, 20, 21–24), page 504 (27, 29, 31, 33, 34)
8.2 Confidence Intervals for a Proportion
Read 499–501

This is the four-step process for calculating a confidence interval. Always do all 4 steps, unless you are instructed to skip portions or do only certain steps. This is also located in the endsheets of your book.

CONFIDENCE INTERVALS: A FOUR-STEP PROCESS

State: What *parameter* do you want to estimate, and at what *confidence level*?

Plan: Identify the appropriate inference *method*. Check *conditions*.

Do: If the conditions are met, perform *calculations*.

Conclude: *Interpret* your interval in the context of the problem.

You can definitely use your calculator to calculate the interval, but know how to use the formula as well. On the AP exam or a test you can be tested on using the formula like this:

Of 700 randomly selected Missouri teens, 61% said they don't believe everything they see online. Which of these represents a 96% confidence interval to estimate the proportion of all Missouri teens who would say that they don't believe everything they see online?

A) $61 \pm 2.054\sqrt{\frac{(61)(39)}{700}}$

B) $61 \pm 2.326\left(\frac{(61)(39)}{\sqrt{700}}\right)$

C) $0.61 \pm 2.054\sqrt{\frac{(0.61)(0.39)}{700}}$

D) $0.61 \pm 2.326\sqrt{\frac{(0.61)(0.39)}{700}}$

E) $0.61 \pm 2.054\left(\frac{(61)(39)}{\sqrt{700}}\right)$

In her social studies class, Kelli learned that Asia makes up 30% of Earth's land area. She wondered if this was really true and asked her dad for help. To investigate, he tossed an inflatable globe to her over and over again, being careful to spin the globe each time. Of the 50 times that her right index finger was pointing to some land mass, her finger was pointing to Asia 16 times. Assume this constitutes a SRS. Construct and interpret a 95% confidence interval for the proportion of Earth's land area comprised by Asia.

Read 501–503

This is the formula for the margin of error for a confidence interval for a proportion:

$$ME = z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Notice that it's the part of the confidence interval formula that was after the ± on the formula sheet.

Solving for the sample size, n:

To choose a value for $\hat{p}$ when solving for the sample size, n, use a recent value from a previous study. If none is available, use 0.5 for $\hat{p}$. When we solve for n, we round the answer up to the next highest number.

Suppose that you wanted to estimate *p* = the true proportion of students at your school who have a tattoo with 96% confidence and a margin of error of no more than 0.12. How many students should you survey?

HW #8 page 505 (35–47 odd)

8.3 Confidence Intervals for a Mean

William S. Gossett

Simulating Confidence Intervals applet
http://www.rossmanchance.com/applets/ConfSim.html

We use a t* critical value for calculating a CI for a population mean because we don't know σ.
(If σ is known, we could use a z* critical value for calculating a CI for a population mean, but we won't encounter that situation.)

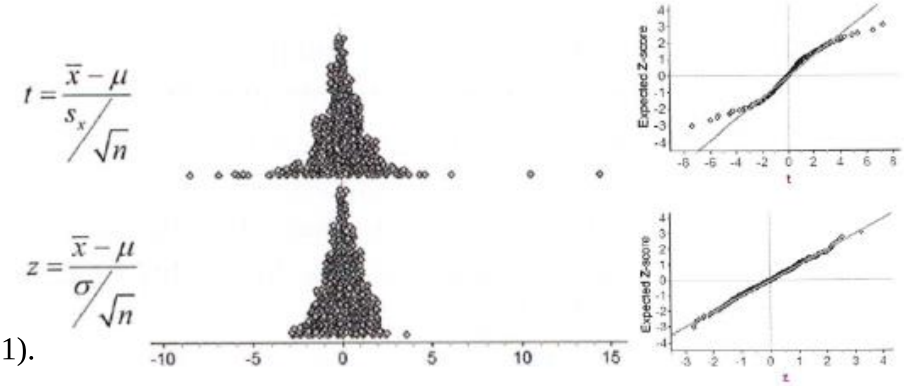
For the situations we will encounter, we can remember when to use t* and when to use z* by remembering:
za$\hat{p}$ta$\bar{x}$, z is appropriate for proportions (categorical data) and t is appropriate for means (quantitative data).

Read 510–514

To calculate the value of *t** to use, use a t-table or invT on the calculator.

For estimating the mean, the "degrees of freedom" is one less than the sample size, or we write df = n-1.

Compare the shape, center, and spread of a t distribution to the standard normal distribution, N(0,1).



Shape: The t distribution is

________________________________ N(0,1).

Center: The center (mean) of the t distribution is

____________________________ N(0,1).

Spread: The t distribution is __________________________________ N(0,1).

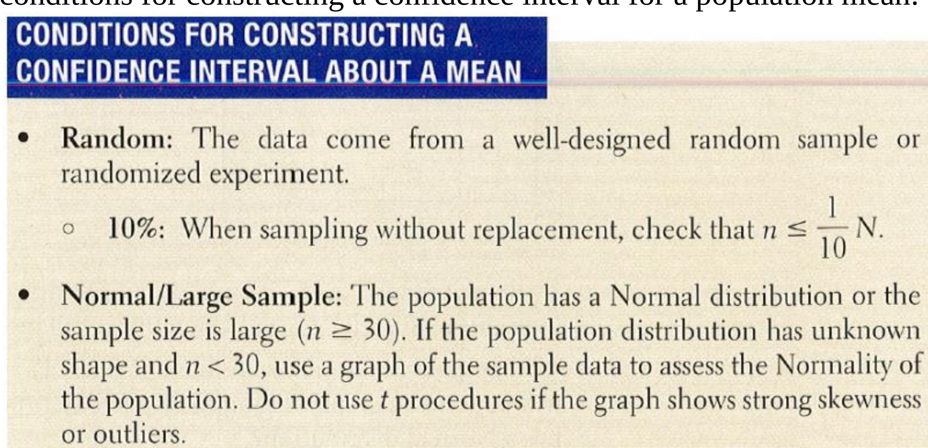
Finding *t** critical values:

(a) Suppose you wanted to construct a 90% confidence interval for the mean μ of a population based on an SRS of size 10. What critical value *t** should you use?

(b) What if you wanted to construct a 99% confidence interval for μ using a sample of size 75?

Read 514–517

These are the three conditions for constructing a confidence interval for a population mean:



CONDITIONS FOR CONSTRUCTING A CONFIDENCE INTERVAL ABOUT A MEAN

- **Random:** The data come from a well-designed random sample or randomized experiment.
  - **10%:** When sampling without replacement, check that $n \leq \frac{1}{10}N$.
- **Normal/Large Sample:** The population has a Normal distribution or the sample size is large ($n \geq 30$). If the population distribution has unknown shape and $n < 30$, use a graph of the sample data to assess the Normality of the population. Do not use *t* procedures if the graph shows strong skewness or outliers.

Read 518–520

This is the formula for the **standard deviation** of the sample mean.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

This one uses σ.

Interpreting the standard deviation:
This is the typical distance of the sample sample mean, x-bar, from the population mean, μ, in repeated SRSs of size n.

This is the formula for the **standard error** of the sample mean.

$$SE_{\bar{x}} = \frac{s}{\sqrt{n}}$$

This one uses s.

Interpreting the standard error:
This is *an estimate of* the typical distance of the sample mean, x-bar, from the population mean, μ, in repeated SRSs of size n.

The standard deviation formula **IS** on the formula sheet.

On the formula sheet, the formula for a confidence interval is in words. Here is the formula specifically for estimating the population mean:

THE ONE-SAMPLE *t* INTERVAL FOR A POPULATION MEAN

When the conditions are met, a C% confidence interval for the unknown mean μ is

$$\bar{x} \pm t^* \frac{s_x}{\sqrt{n}}$$

where t^* is the critical value for the t_{n-1} distribution, with C% of the area between $-t^*$ and t^*.

A student, who was concerned about how heavy his textbooks were, wanted to estimate the average weight of student backpacks loaded with 7 classes worth of books. He selected a random sample of 36 students from the school and asked them to put all their textbooks in their backpacks. The student weighed each student's backpack (in pounds). The mean weight was $\bar{x}$ = 30.14 pounds with a standard deviation of s_x = 2.16 pounds.

(a) Construct and interpret a 95% confidence interval for the mean weight of a loaded backpack.

(b) Some pediatricians recommend that backpacks should not weigh over 20% of a student's body weight. Thus, if the average high school student weight is 130, student backpack weight should not exceed 26 pounds. Does the interval in part (a) provide convincing evidence that the average weight of loaded high school backpacks is too heavy? Explain.

HW #9: page 506 (49–52), page 527 (55, 57, 59, 61, 65, 67)

8.3 Confidence Intervals for a Mean

Read 520–522

Common error for the Normal/Large Sample condition: Forgetting to link the rule to the problem.

Writing this shows only that you know the rule:	Writing this shows that you know the rule AND that the rule applies:
$n \geq 30$.	$n=36$ and $36 \geq 30$.
This isn't enough to earn credit.	This earns you credit.

If you think the Normal/Large Sample condition isn't met, identify the problem or concern and then write "I will proceed with caution."

You can definitely use your calculator for the Do step, but be sure to identify the numbers you enter into the calculator.

A student randomly selected 18 rolls of a generic brand of paper towels to measure how strong the towels were when soaked with water. To do this, she counted how many marbles could be suspended in a randomly selected soaked paper towel from each roll before the towel broke. Here are the results from her 18 rolls:

1	7	8	9	9	10	10	11	11
12	12	12	13	13	13	14	16	16

Construct and interpret a 99% confidence interval for μ = the mean number of marbles that this generic brand of paper towels can hold, without breaking, when wet.

Read 522–524

When you are asked to find the minimum sample size, start with the margin of error formula: $ME = t^* \frac{\sigma}{\sqrt{n}}$ and then we have to change t* to z* (since we don't know the sample size, n, to find t*) and we have to use the sample standard deviation, σ (since we don't know the population standard deviation, σ).

That means we will use this altered formula: $ME = z^* \frac{s}{\sqrt{n}}$

The school newspaper staff wants to estimate how much students spend on prom, on average. They want to estimate μ at the 90% confidence level with a margin of error of at most $30. A pilot study indicated that the standard deviation is about $60. How many students need to be surveyed?

HW #10: Page 528 (69, 71, 73, 75–78)
Chapter 8 Review/FRAPPY!
Frappy page 530
HW #11: Page 532 Chapter review exercises
Chapter 8 Review
HW #12: page 534 AP Practice Test
Chapter 8 Test