Algebra 2

Unit 5 (Day 1) Graphing Rational Functions

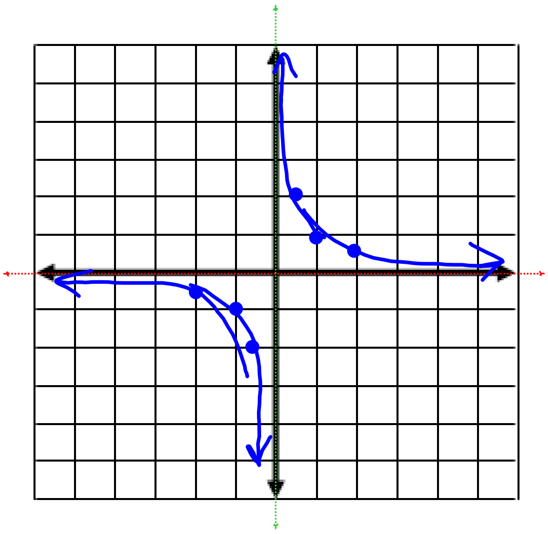
Objectives: Graph rational functions in vertex form.
Identify zeros, vertical and horizontal asymptotes.

The function $f(x) = \frac{1}{x}$ has a vertical asymptote at $x = 0$ and a horizontal asymptote at $y = 0$.

x	y
1	1
2	$\frac{1}{2}$
$\frac{1}{2}$	2

x	y
-1	-1
-2	$-\frac{1}{2}$
$-\frac{1}{2}$	-2

horizontal asymptote
(what y can't be equal to)



vertical asymptote
(what x can't be equal to)

The function $f(x) = \frac{1}{x}$ can be transformed according to the following rules:

$|a| \longrightarrow$ vertical stretch or compression factor
$a < 0 \longrightarrow$ reflection across the x-axis

$k \longrightarrow$ vertical translation
down for $k < 0$; up for $k > 0$

$$f(x) = \frac{a}{x-h} + k$$

+3 up 3
−3 down 3

$h \longrightarrow$ horizontal translation
left for $h < 0$; right for $h > 0$

x − 2
right 2

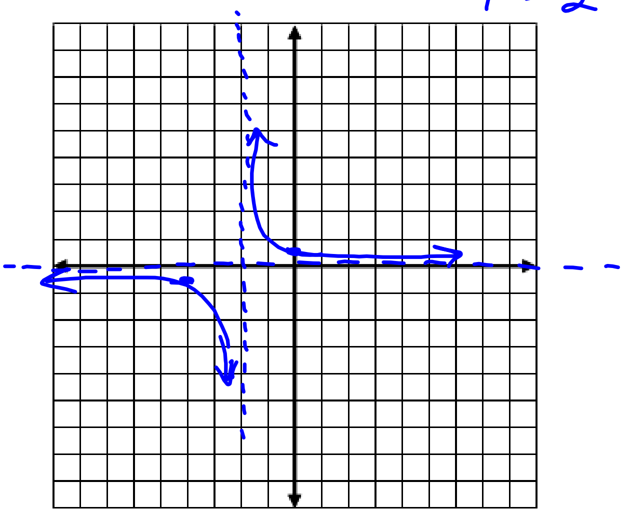
x + 2
left 2

Example 1: Using the graph of $f(x) = \frac{1}{x}$ as a guide, describe the transformation and graph each function.

a. $g(x) = \frac{1}{(x+2)}$

left 2

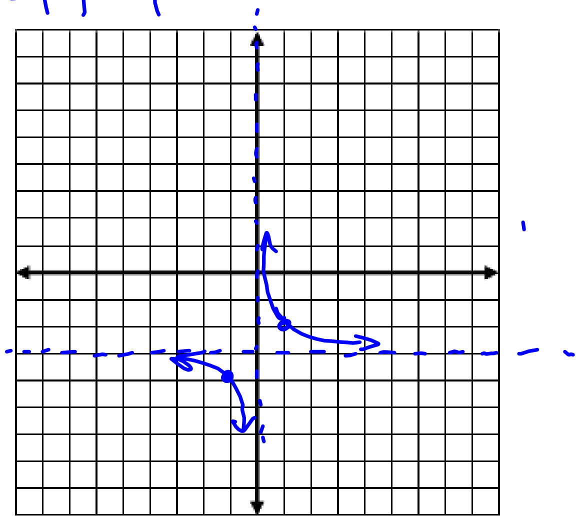
x	y
0	$\frac{1}{2}$
-4	$-\frac{1}{2}$



b. $g(x) = \frac{1}{x} - 3$

down 3

x	y
1	-2
-1	-4



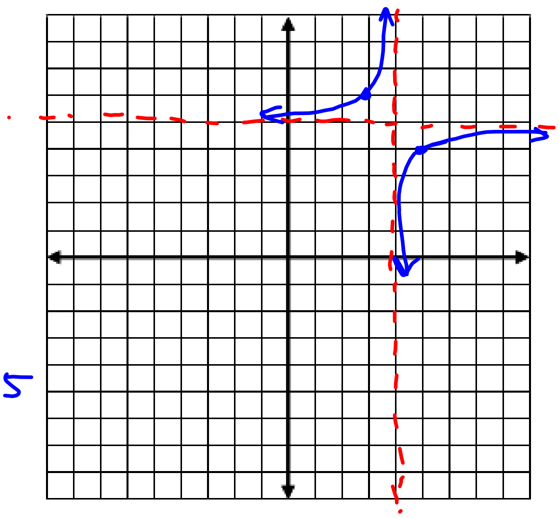
c. $g(x) = \frac{1}{4-x} + 5$

$= \frac{1}{-1(x-4)} + 5$

reflect over H.A.

$= \frac{-1}{1(x-4)} + 5$

right 4 up 5



x	y	
5	4	$\frac{1}{-1} + 5$
3	6	$\frac{1}{1} + 5$

Rational Functions

For a rational function of the form $f(x) = \frac{a}{x - h} + k$,
- the graph is a hyperbola.
- there is a vertical asymptote at the line $x = h$, and the domain is $\{x \mid x \neq h\}$. $(-\infty, h)$ or (h, ∞)
- there is a horizontal asymptote at the line $y = k$, and the range is $\{y \mid y \neq k\}$. $(-\infty, k)$ or (k, ∞)

Example 2: Identify the asymptotes, domain, and range of each function.

a. $g(x) = \frac{1}{x+3} - 2$

$x + 3 \neq 0$
$x \neq -3$

Vertical: $x = -3$

Horizontal: $y = -2$

Domain: $\{x \mid x \neq -3\}$

Range: $\{y \mid y \neq -2\}$

D: $(-\infty, -3)$ or $(-3, \infty)$

R: $(-\infty, -2)$ or $(-2, \infty)$

b. $g(x) = \frac{1}{x-3} - 5$

down 5

Vertical: $x = 3$

Horizontal: $y = -5$

Domain: $\{x \mid x \neq 3\}$

Range: $\{y \mid y \neq -5\}$

Zeros and Vertical Asymptotes — Rational Functions

If $f(x) = \frac{p(x)}{q(x)}$, where p and q are polynomial functions in standard form with no common factors other than 1, then the function f has

- zeros at each real value of x for which $p(x) = 0$. numerator = 0
- a vertical asymptote at each real value of x for which $q(x) = 0$. denom. is 0

Horizontal Asymptotes — Rational Functions

Let $f(x) = \frac{p(x)}{q(x)}$, where p and q are polynomial functions in standard form with no common factors other than 1. The graph of f has at most one horizontal asymptote.

- If degree of $p >$ degree of q, there is no horizontal asymptote. $\frac{x^2+4}{x-2}$
- If degree of $p <$ degree of q, the horizontal asymptote is the line $y = 0$. $\frac{x-2}{x^2-9}$ $\frac{5}{12000}$
- If degree of $p =$ degree of q, the horizontal asymptote is the line

$$y = \frac{\text{leading coefficient of } p}{\text{leading coefficient of } q}.$$

$\frac{x^1-4}{2x^1-3} \rightarrow y = \frac{1}{2}$

$\frac{4x^2+9}{2x^2-5} \rightarrow y = \frac{4}{2}$

$y = 2$

Holes in Graphs — Rational Functions

If a rational function has the same factor $x - b$ in both the numerator and the denominator, then there is a hole in the graph at the point where $x = b$, unless the line $x = b$ is a vertical asymptote.

Example 3: Identify all zeros, horizontal and vertical asymptotes and holes in the graph of the function.

a. $f(x) = \dfrac{x^2 + x - 6}{x + 4} = \dfrac{(x-2)(x+3)}{x+4}$

$x - 2 = 0$ $x + 3 = 0$
$x = 2$ $x = -3$

x that makes num 0
Zeros: $x = 2,\ x = -3$

x = # that makes denom. 0 but not numerator
Vertical: $x = -4$

$x + 4 = 0 \rightarrow x = -4$

Horizontal: none num deg > denom deg

b. $f(x) = \dfrac{x - 1}{x^2 - 4} = \dfrac{x-1}{(x+2)(x-2)}$

denom deg > num deg

$x - 1 = 0 \rightarrow$
Zeros: $x = 1$

$x + 2 = 0$ $x - 2 = 0 \rightarrow$
Vertical: $x = -2$ $x = 2$

Horizontal: $y = 0$

c. $f(x) = \frac{6x - 12}{3x + 4}$ same degree

$6x - 12 = 0 \rightarrow$

Zeros: $x = 2$

$3x + 4 = 0 \rightarrow$

Vertical: $x = -\frac{4}{3}$ or $-1\frac{1}{3}$

Horizontal: $y = \frac{6}{3} \rightarrow y = 2$

d. $f(x) = \frac{x^2 - 7x + 12}{x^2 - 9}$

Zeros: ____________________

Vertical: __________________

Horizontal: ________________

Assignment: Worksheet 8.4 (Day 1)

$(x+1)\cancel{(x-1)}$

Q: What's the difference between Tiger Woods and $f(x)=\frac{x^2-1}{\cancel{x-1}}$?

A: One gets a hole in 1, and the other has a hole at 1.

hole at

$x-1=0$

$x=1$

$f(x)=x+1$

$y=x+1$

$y=mx+b$