

Algebra 2

5.1 (Day 2) Graphing Rational Functions

Objective: Graph rational functions.Review:

zeros - values that make the numerator equal zero.

vertical asymptotes - values that make the denominator equal zero.

horizontal asymptotes - degree of $N > D$: none, degree of $N < D$: $y = 0$,
degree of $N = D$: divide leading coefficientsExample 1: Identify the zeros and asymptotes of each function, then graph.

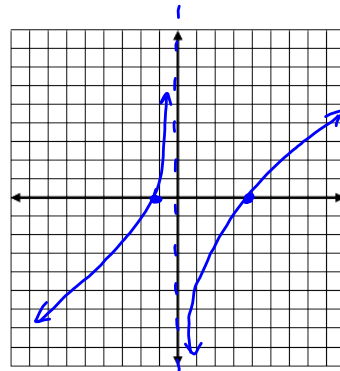
a. $f(x) = \frac{x^2 - 3x - 4}{x}$

$(x+1)(x-4)$

$N=0 \quad x+1=0 \quad x-4=0$
Zeros: $x = -1 \quad x = 4$

$D=0 \quad x=0$
Vertical: $x = 0$

$N \text{ deg} > D \text{ deg}$
Horizontal: none



b. $f(x) = \frac{x-2}{x^2-1}$

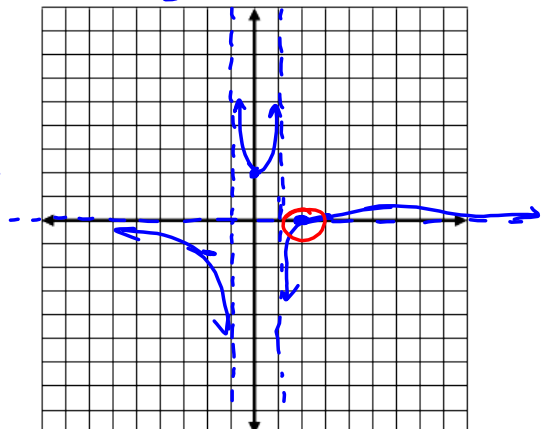
$N=0 \quad x-2=0$
Zeros: $x = 2$

$D=0 \quad x^2-1=0 \rightarrow (x+1)(x-1)=0$
Vertical: $x = -1 \quad x = 1$

$\text{num deg} < \text{denom deg}$
Horizontal: $y = 0$

$x = -2 \quad \frac{-4}{4-1} = \frac{-4}{3}$

$x = 3 \quad \frac{3-2}{9-1} = \frac{1}{8}$

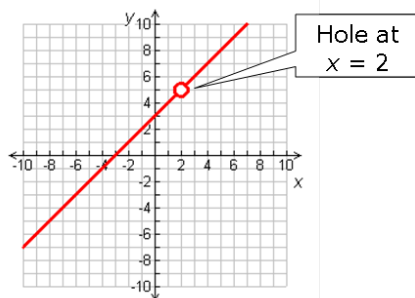
Note: If a solution lies on a horizontal asymptote, then the graph of the rational function will cross the horizontal asymptote at that point.

In some cases, both the numerator and the denominator of a rational function will equal 0 for a particular value of x . As a result, the function will be undefined at this x -value. If this is the case, the graph of the function may have a **hole**. A **hole** is an omitted point in a graph.

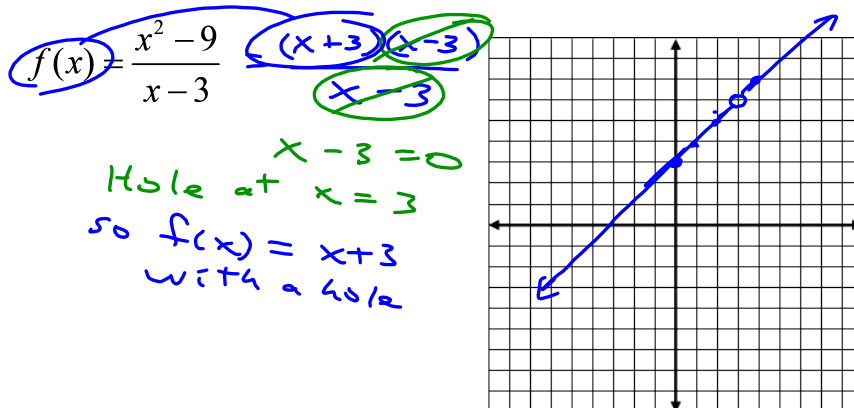
Holes in Graphs Rational Functions

If a rational function has the same factor $x - b$ in both the numerator and the denominator, then there is a hole in the graph at the point where $x = b$, unless the line $x = b$ is a vertical asymptote.

Example:



Example 2: Identify **holes** in the graph of the function. Then graph.



Ex. 3 Given the parent function, $f(x) = 1/x$, write a function $g(x)$ that has a graph with a vertical asymptote at $x = 1$, a horizontal asymptote at $y = -2$, and passes through the point $(0, -3)$

Write your answer in the form $f(x) = \frac{a}{x-h} + k$

VA $x = 1 \rightarrow$ denom is

$$\frac{-1}{x-1} = 0$$

HA $y = -2$
 $y = k$

$$f(x) = \frac{a}{x-1} - 2$$

$$-3 = \frac{a}{0-1} - 2$$

$$-3 = \frac{a}{-1} - 2$$

$$-3 = -a - 2$$

$$+2 \quad +2$$

$$-1 = -a$$

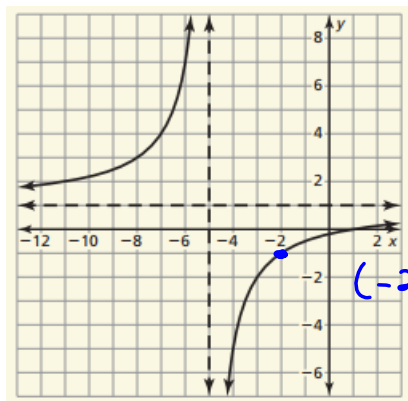
$$a = 1$$

$$f(x) = \frac{1}{x-1} - 2$$

Ex. 4 Write the equation of the rational function in the form $f(x) = \frac{a}{x-h} + k$.

VA = $x = -5$
(left + 5)

HA $y = 1$
up 1



$$f(x) = \frac{a}{x - (-5)} + 1$$

$$f(x) = \frac{a}{x+5} + 1$$

$$-1 = \frac{a}{-2+5} + 1$$

$$(-2, -1) \quad -1 = \frac{a}{3} + 1$$

$$-1 - 1 = \frac{a}{3} - 1$$

$$-2 = \frac{a}{3}$$

$$-6 = a$$

$$f(x) = \frac{-6}{x+5} + 1$$