

1.1 Solving and Writing Linear Equations

Objectives:

Solve and write linear equations using a variety of methods.

An **equation** is a mathematical statement that two expressions are equivalent. The **solution set of an equation** is the value or values of the variable that make the equation true.

A **linear equation in one variable** can be written in the form $ax = b$, where a and b are constants and $a \neq 0$.

Linear Equations in One variable

$$4x = 8$$

$$3x - \frac{2}{3}x = -9$$

$$2x - 5 = 0.1x + 2$$

Nonlinear Equations

$$3\sqrt{x} + 1 = 32$$

$$\frac{2}{x^2} + 1 = 41$$

$$3 - 2^x = -5$$

Solving a linear equation requires isolating the variable on one side of the equation by using the properties of equality.

To isolate the variable, perform the inverse or opposite of every operation in the equation on both sides of the equation.

Do inverse operations in the reverse order of operations.

$$+ \leftrightarrow -$$

$$\sqrt{x} \leftrightarrow x^2$$

$$\div \leftrightarrow \times$$

Ex. 1 Solve $4(m + 12) = -36$

$$\begin{array}{r} 4m + 48 = -36 \\ -48 \quad -48 \end{array}$$

$$\frac{4m}{4} = \frac{-84}{4}$$

$$m = -21$$

$$\frac{4(m+12) = -36}{4} = \frac{-36}{4}$$

$$\begin{array}{r} m+12 = -9 \\ -12 \quad -12 \end{array}$$

Ex. 2 Solve $3k - 14k + 25 = 2 - 6k - 12$ $m = -21$

$$\begin{array}{r} -11k + 25 = 2 - 6k - 12 \\ +14k \quad +14k \\ 3k + 25 = 2 - 6k + 14k - 12 \\ -3k \quad -3k \end{array}$$

$$25 = 2 + 5k - 12$$

$$\begin{array}{r} 25 = 5k - 10 \\ +10 \quad +10 \end{array}$$

$$\frac{35}{5} = \frac{5k}{5}$$

$$7 = k$$

Ex. 3 Solve $3(w + 7) - 5w = w + 12$.

$$3w + 21 - 5w = w + 12$$

$$-2w + 21 = w + 12$$

$$-w - 21 = -4 - 21$$

$$\frac{-3w}{-3} = \frac{-9}{-3} \quad w = 3$$

Ex. 4 Solve $3v - 9 - 4v = -(5 + v)$.

$$-v - 9 = -5 - v$$

$$-9 = -5 \quad \text{no solution}$$

An equation that is true for all values of the variable, such as $x = x$, is an **identity**. An equation that has no solutions, such as $3 = 5$, is a **contradiction** because there are no values that make it true.

Ex. 5 Solve $2(x - 6) = -5x - 12 + 7x$.

$$2x - 12 = 2x - 12$$

$$-12 = -12 \quad \text{infinite \# of solutions}$$

Ex. 6 Write and solve a linear equation.

The local phone company charges \$12.95 a month for the first 200 of air time, plus \$0.07 for each additional minute. If Nina's bill for the month was \$14.56, how many additional minutes did she use?

$$\begin{array}{l}
 x = \text{Nina's } \overset{\text{total}}{\text{additional}} \text{ minutes} \\
 \text{total - included} \\
 12.95 + .07(x - 200) = 14.56 \\
 \underline{12.95} + .07x - \underline{14} = 14.56 \\
 -1.05 + .07x = 14.56 \\
 +1.05 \qquad \qquad \qquad +1.05 \\
 \underline{.07x} = \underline{15.61} \\
 \underline{.07} \qquad \qquad \underline{.07} \\
 x = 223 \text{ } \overset{\text{total}}{\text{additional}} \text{ minutes} \\
 223 - 200 = 23 \text{ additional}
 \end{array}$$

Ex. 7 Write and solve a linear equation.

/ Stacked cups are to be placed in a pantry. One cup is 3.25 in. high and each additional cup raises the stack 0.25 in. How many cups fit between two shelves 14 in. apart?

$$\begin{array}{l}
 x = \# \text{ of cups} \\
 3.25 + .25(x - 1) = 14 \\
 3.25 + .25x - .25 = 14 \\
 3 + .25x = 14 \\
 -.25x = 11 \\
 x = 44
 \end{array}
 \quad \left| \quad
 \begin{array}{l}
 x = \# \text{ of cups} \\
 \text{after the 1st one} \\
 3.25 + .25x = 14 \\
 -3.25 \qquad \qquad -3.25 \\
 \underline{.25x} = \underline{10.75} \\
 \underline{.25} \qquad \qquad \underline{.25} \\
 x = 43 \\
 \text{total} = 43 + 1 \\
 = 44
 \end{array}$$