Algebra 2

1.6 Solving Linear Systems in Three Variables

<u>Objectives</u>:
- Solve systems of equations in three variables using substitution and elimination.
- Solve real-life problems.

Systems of three equations with three variables are often called 3 x 3 systems. In general, to find a single solution to any system of equations, you need as many equations as you have variables.

The solution to a system of three equations in three variables is an **<u>ordered triple</u>**, (x, y, z), whose coordinates make each equation true.

We can solve 3 x 3 systems of using a combination of thesubstitution and elimination methods.

Ex. 1: Solve.

• Check solutions in <u>all 3</u> equations!

$x + 2y + z = 4$

$4y - 3z = 1 \rightarrow \cancel{4y} - 3z = 1$

$y + 5z = 6 \xrightarrow[\cdot(-4)]{} \cancel{-4y} - 20z = -24$

$-23z = -23$

$z = 1$

$y + 5z = 6$

$y + 5(1) = 6$

$y + 5 = 6$

$y = 1$

$x + 2y + z = 4$

$x + 2(1) + 1 = 4$

$x + 3 = 4$

$x = 1$

$(1, 1, 1)$

Ex. 2: Solve.

$x + 2z = 5$
$-4x + 3y = 0$
$y - 2z = 9$

$x + 2z = 5$
$y - 2z = 9$
$x + y = 14$

$\cdot(4)$

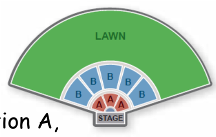
$-4x + 3y = 0$
$4x + 4y = 56$
$7y = 56$
$y = 8$

$x + y = 14$
$x + 8 = 14$
$x = 6$

$x + 2z = 5$
$6 + 2z = 5$
$-6 \quad -6$
$2z = -1$
$z = -\frac{1}{2}$

$(6, 8, -\frac{1}{2})$

Ex. 3: Write a system of equations and solve.



An amphitheater charges \$75 for each seat in Section A, \$55 for each seat in Section B, and \$30 for each lawn seat. There are three times as many seats in Section B as in Section A. The revenue from selling all 23,000 seats is \$870,000. How many seats are in each section of the amphitheater?

- 1st Define the variables!

A = # of sec. A seats
B = # " " B "
L = # " lawn "

$B = 3A$

$A + B + L = 23000 \xrightarrow{B=3A} A + 3A + L = 23000$

$75A + 55B + 30L = 870000 \rightarrow 75A + 55(3A) + 30L = 870000$
$165A$

$4A + L = 23000 \xrightarrow{\cdot(-60)} -240A - 60L = -1380000$
$240A + 30L = 870000 \rightarrow 240A + 30L = 870000$

$-30L = -510000$
$\frac{-30L}{-30} = \frac{-510000}{-30}$
$L = 17000$

$4A + L = 23000$
$4A + 17000 = 23000$
$-17000 \quad -17000$
$4A = 6000$
$A = 1500$

$B = 3A$
$B = 3(1500) = 4500$

To solve a three-variable system using elimination:

(Triple) Elimination - use when all 3 equations have all 3 variables

1. Put 2 of the equations together an eliminate a variable.
2. Use the 3rd equation and one of the other original equations to eliminate the **same variable**.
3. Put the 2 resulting equations together and eliminate one of the two variables.
4. Solve the resulting equation.
5. Substitute the answer to solve for the 2nd variable.
6. Substitute both answers into one of the original equations to solve for the 3rd variable.

Ex. 4: Solve.

$4x + 3y + 2z = 34$
$2x + 4y + 3z = 45$
$3x + 2y + 4z = 47$

$\cdot(-2)$
$-8x - 6y - 4z = -68$
$3x + 2y + 4z = 47$
$-5x - 4y = -21$

① $\cdot 3$
② $\cdot(-2)$

$12x + 9y + 6z = 102$
$-4x - 8y - 6z = -90$
$8x + y = 12$
$-8x \quad -8x$
$y = 12 - 8x$

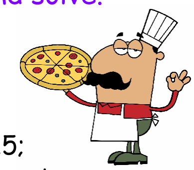
$-5x - 4(12 - 8x) = -21$
$-5x - 48 + 32x = -21$
$+48 \quad +48$
$27x = 27$
$x = 1$

$y = 12 - 8x$
$y = 12 - 8(1)$
$y = 12 - 8$
$y = 4$

$4x + 3y + 2z = 34$
$4 \cdot 1 + 3 \cdot 4$
$4 + 12 + 2z = 34$
$16 + 2z = 34$
$-16 \quad -16$
$2z = 18$
$z = 9$

$(1, 4, 9)$

Ex. 5: Write a three-variable system of equations and solve.



Three orders are placed at a pizza shop.
Two small pizzas, a liter of soda (drink), and a salad cost \$14;
one small pizza, a liter of soda, and three salads cost \$15;
and three small pizzas, a liter of soda, and two salads cost \$22.
How much does each item cost?

① $2p + D + S = 14$
② $p + D + 3S = 15$
③ $3p + D + 2S = 22$

① − ② $p - 2S = -1$
③ − ① $p + S = 8$
subtract
$-3S = -9$
$S = 3$

$p + S = 8$
$p + 3 = 8$
$p = 5$

$2p + D + S = 14$
$2(5) + D + 3 = 14$
$10 + D + 3 = 14$
$D + 13 = 14$
$D = 1$

Assignment: Worksheet 1.6

Teacher: Why is your homework paper blank?

Student: