

Algebra 2

Compound Inequalities

Objectives:

- Represent intervals using interval notation.
- Write, solve and graph compound inequalities.
- Use compound inequalities to solve real-life problems.

Interval Notation is a notation used to represent an interval on a number line as a pair of numbers. We will use interval notation to state solutions for inequalities.



In interval notation, the symbols [] are used to include an endpoint in an interval, and the symbols () are used to exclude an endpoint from an interval.

Recall: When graphing on a number line, we use...

() ○ open circle if not equal to, < or >
 [] ● closed circle if equal to, ≤ or ≥

Examples:

<u>Inequality</u>	<u>Interval Notation</u>	<u>Graph</u>
$3 < x < 5$	$(3, 5)$	
$3 \leq x \leq 5$	$[3, 5]$	
$3 < x \leq 5$	$(3, 5]$	
$3 \leq x < 5$	$[3, 5)$	

- An interval that extends forever in the **positive direction** goes to **infinity** (∞).
- An interval that extends forever in the **negative direction** goes to **negative infinity** ($-\infty$).

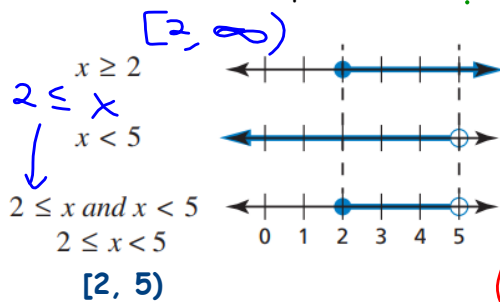
Because these symbols do not represent real numbers, they are always enclosed by a parenthesis.

More Examples:

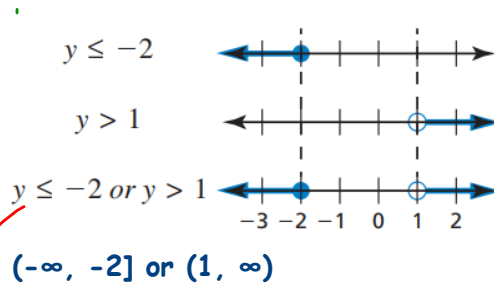
Inequality	Interval Notation	Graph
$x > 3$	$(3, \infty)$	
$x \geq 3$	$[3, \infty)$	
$x < 5$	$(-\infty, 5)$	
$x \leq 5$	$(-\infty, 5]$	
	$(-\infty, \infty)$	

A **compound inequality** is an inequality formed by joining two inequalities with the word "and" or the word "or."

The graph of a compound inequality with "and" is the intersection of the graphs of the inequalities. The graph shows numbers that are solutions of both inequalities.



The graph of a compound inequality with "or" is the union of the graphs of the inequalities. The graph shows numbers that are solutions of either inequality.



~~$-2 \leq y \text{ or } y > 1$~~
 ~~$-2 \leq y < 1$~~

Solve and graph each compound inequality.
Write the answer in interval notation.

REMEMBER: A compound inequality with "and" can be written as a single inequality. For example, you can write $x > -8$ and $x \leq 4$ as $-8 < x \leq 4$.

Ex. 1: $-3 < -2x + 1 \leq 9$

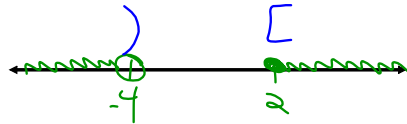
$$\begin{array}{l} -1 \quad -1 \quad -1 \\ -4 < -2x \leq 8 \\ \frac{-4}{-2} < \frac{-2x}{-2} \leq \frac{8}{-2} \\ 2 > x \geq -4 \end{array}$$

$$-4 \leq x < 2$$

$$[-4, 2)$$

Ex. 2: $2x + 1 < -7$ or $3 - 2x \leq -1$

$$\begin{array}{l} -1 \quad -1 \quad -3 \quad -3 \\ 2x < -8 \quad \text{or} \quad -2x \leq -4 \\ \frac{2x}{2} < \frac{-8}{2} \quad \text{or} \quad \frac{-2x}{-2} \leq \frac{-4}{-2} \\ x < -4 \quad \text{or} \quad x \geq 2 \end{array}$$

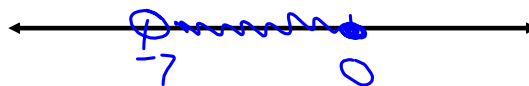


$$(-\infty, -4) \text{ or } [2, \infty)$$

Ex. 3: $14x + 9 < 15x + 16$ and $16 - 4x \geq 18x + 16$

$$\begin{array}{l} -15x \quad -9 \quad -15x \quad -9 \quad -16 \quad -18x \quad -18x \quad -16 \\ -x < 7 \quad \text{and} \quad -22x \geq 0 \\ \frac{-x}{-1} < \frac{7}{-1} \quad \text{and} \quad \frac{-22x}{-22} \geq \frac{0}{-22} \\ x > -7 \quad \text{and} \quad x \leq 0 \end{array}$$

$$-7 < x \leq 0$$



$$(-7, 0]$$

Ex. 4: Write and solve a compound inequality.

Jill wants to know what she needs to score on her next Algebra 2 test. The scores for her tests are 75, 82, 78 and 85. What does Jill need to score on her next test if she wants her average to be between 80 and 85?

$$80 < \frac{75+82+78+85+x}{5} < 85$$

$$5 \cdot 80 < \frac{320+x}{5} < 85 \cdot 5$$

$$400 < 320+x < 425$$
$$\begin{array}{r} -320 \quad -320 \quad -320 \\ \hline \end{array}$$

$$80 < x < 105$$

Jill needs b/w 80 & 105
on the next test.