Algebra 2

1.8 Absolute Value Equations and Inequalities

The **ABSOLUTE VALUE** of a number x, written |x |, is

the distance from 0.

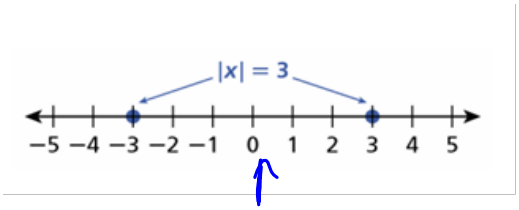
Since it is a distance, the absolute value is always considered POSITIVE.

Absolute-value equations and inequalities can be represented by compound statements.

Consider the equation |x| = 3. → $|x - 0| = 3$

This means all the numbers 3 UNITS FROM ZERO.

So, x = 3 or x = −3.



Ex. 1 What are the possible solutions?

$|x| = 5$

$x = 5$ or $x = -5$

$|x| = 0$

$x = 0$

$|x| = -5$

no solution

abs. value of a # can't end up negative.

When solving absolute value equations:

Isolate the absolute value.
Write two related equations, if possible.
Solve and check your solution.

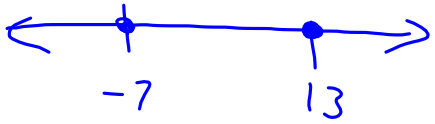
Ex 2) Solve.

$|-3 + k| = 10$

$-3 + k = 10$ or $-3 + k = -10$

$+3 \quad +3 \qquad +3 \quad +3$

$k = 13$ $\qquad$ $k = -7$



Ex 3) Solve.

$\left|\frac{x}{4}\right| + 6 = 2$

$-6 \quad -6$

$\left|\frac{x}{4}\right| = -4$

no solution

Ex 4) Solve.

$|2x + 5| = 0$

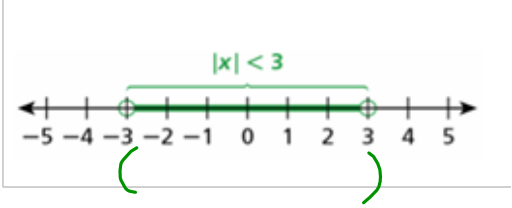
$2x + 5 = 0$

$-5 \quad -5$

$2x = -5$

$x = -\frac{5}{2}$

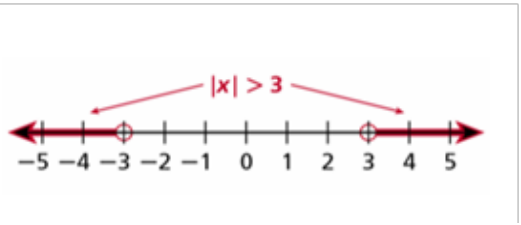
Consider $|x| < 3$, this means
all the #s that are less than 3 units from 0



$x > -3$ and $x < 3$

We can write this as a conjunction, using "and",
or in interval notation as $(-3, 3)$.

Consider $|x| > 3$, this means
all the #s more than 3 units from 0.



We can write this as a DISjunction $x < -3$ or $x > 3$

or using interval notation $(-\infty, -3)$ or $(3, \infty)$.

Solving an Absolute-value Inequality
1. Isolate the absolute-value expression, if necessary.
2. Rewrite the absolute-value expression as a compound inequality.
3. Solve each part of the compound inequality for x.

Helpful Hint

Think: Greator inequalities involving > or ≥ symbols are disjunctions.

Think: Less thand inequalities involving < or ≤ symbols are conjunctions.

$|x-m| > \geq$ or

$|x-m| < \leq$ and

A B C .... M N O P ... Z

Solve. Graph your solution.

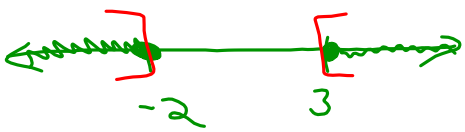
Ex. 5 $|-4q + 2| \geq 10$

$-4q+2 \geq 10$ or $-4q+2 \leq -10$

$-2 \quad -2 \qquad -2 \quad -2$

$\frac{-4q}{-4} \geq \frac{8}{-4} \qquad \frac{-4q}{-4} \leq \frac{-12}{-4}$

$q \leq -2 \qquad q \geq 3$



$(-\infty, -2] \text{ or } [3, \infty)$

Ex. 6 $|0.5r| - 3 \geq -3$

$+3 \quad +3$

$|.5r| \geq 0$

$\frac{.5r}{.5} \geq \frac{0}{.5}$ or $\frac{.5r}{.5} \leq \frac{0}{.5}$

$r \geq 0$ or $r \leq 0$

$(-\infty, \infty)$

Ex. 7 $\frac{|2x+7|}{3} \le 1$ ·3

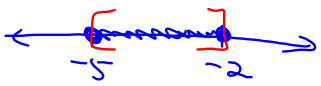
$|2x+7| \le 3$

$2x+7 \le 3$ and $2x+7 \ge -3$

$-7 \quad -7 \qquad -7 \quad -7$

$\frac{2x}{2} \le \frac{-4}{2}$ $\qquad \frac{2x}{2} \ge \frac{-10}{2}$

$x \le -2$ and $x \ge -5$



$[-5, -2]$

Ex. 8 $-\frac{1}{2}|p-2| \ge 3$ (-2)

$|p-2| \le -6$

no solution

but...

$|p-2| \ge -6$

$(-\infty, \infty)$