Algebra 2

2.3 Writing Quadratic Functions in Vertex Form

Objective: Write quadratic functions in vertex form.

☆ Perfect Square Trinomials:

$a^2 + 2ab + b^2 = (a + b)^2$

$a^2 - 2ab + b^2 = (a - b)^2$

Ex. $x^2 - 8x + 16$

$(x-4)(x-4)$

Check:

$x^2 - 4x - 4x + 16$

$x^2 - 8x + 16$

16

-8

Check

$(x+4)(x-4)$

$x^2 \cancel{-4x+4x} - 16$

$x^2 - 16$

Sometimes you need to add a term to an expression $x^2 + bx$ to make it a perfect square trinomial. This process is called **completing the square**.

Completing the Square

To complete the square for the expression $x^2 + bx$, add $\left(\frac{b}{2}\right)^2$.

$$x^2 + bx + \left(\frac{b}{2}\right)^2 \ \textit{factors as} \ \left(x + \left(\frac{b}{2}\right)\right)^2$$

Ex. Complete the square for each expression.
Write the result as a binomial squared.

a. $x^2 + 14x +$ 49

$\frac{1}{2}$ □

$(x+7)^2$

b. $x^2 - 12x +$ 36

$\frac{1}{2}$ □

$(x-6)^2$

Writing Quadratic Equations in Vertex Form

Standard form: $f(x) = ax^2 + bx + c$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Axis of Symmetry: $x = \frac{-b}{2a}$

Vertex form: $f(x) = a(x - h)^2 + k$

Vertex: (h, k)

We can use Completing the Square to rewrite any quadratic function in vertex form.

Steps for Completing the Square for Vertex Form

1. Move the constant term to the right of = sign, (with the y).
2. Factor out the leading coefficient of x^2 (if necessary).
3. Take $\frac{1}{2}$ the coefficient of the x term, square it and add it to both sides.
4. Simplify the right side of the equation.
5. Factor left side as a Perfect SquareTrinomial.
6. Solve for y.

Ex. 1: Write each function in vertex form, and identify its vertex.

Check your answers using the equation for the axis of symmetry!

a. $f(x) = x^2 + 16x - 12 + 64 - 64$

$\frac{1}{2}$ $\square$

$f(x) = (x+8)^2 - 76$

$V(-8, -76)$

$x = \frac{-b}{2a} = \frac{-16}{2(1)} = -8$

b. $f(x) = x^2 - 8x + 18 + 16 - 16$

$f(x) = (x-4)^2 + 2$

$V(4, 2)$

$x = \frac{-b}{2a} = \frac{8}{2(1)} = 4$

c. $f(x) = 3x^2 - 18x + 7$

$f(x) = 3(x^2 - 6x + 9) + 7 - 27$

$\frac{1}{2}$ $\square$

$f(x) = 3(x-3)^2 - 20$

$V(3, -20)$

$x = \frac{-b}{2a} = \frac{18}{2(3)} = \frac{18}{6} = 3$

d. $f(x) = 2x^2 - 6x - 8$

$f(x) = 2(x^2 - 3x + \frac{9}{4}) - 8 - \frac{\cancel{18}^{9}}{\cancel{4}_{2}}$

$f(x) = 2(x - \frac{3}{2})^2 - \frac{25}{2}$

$\frac{-8 \cdot 2}{1 \cdot 2} - \frac{9}{2}$

$\frac{-16}{2} - \frac{9}{2}$

$x = \frac{-b}{2a} = \frac{6}{2 \cdot 2} = \frac{6}{4} = \frac{3}{2}$

$V(\frac{3}{2}, \frac{-25}{2})$

Ex. 2: The height y (in feet) of a baseball t seconds after it is hit can be modeled by the function $y = -16t^2 + 96t + 3$.

$y = -16(t^2 - 6t + 9) + 3 + 144$

a. Find the maximum height of the baseball.

(Find the vertex.)

y part of vertex

$y = -16(x-3)^2 + 147$

147 feet

b. How long does the ball take to hit the ground?

(Set equal to 0 and solve.)

$0 = -16t^2 + 96t + 3$

$$t = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-96 \pm \sqrt{96^2 - 4(-16)(3)}}{2(-16)}$$

$$= \frac{-96 \pm \sqrt{9216 + 196}}{-32} = \frac{-96 \pm \sqrt{9412}}{-32}$$

$$= \frac{-96 \pm \cancel{97.015}\ 97}{-32} = \frac{-193}{-32} = 6.03$$

about 6 seconds.

Ex. 3: Convert $y = 2(x-1)^2 + 3$ into $ax^2 + bx + c$ form.

$(x-1)(x-1)$

$y = 2(x^2 - 2x + 1) + 3$

$y = 2x^2 - 4x + 2 + 3$

$y = 2x^2 - 4x + 5$