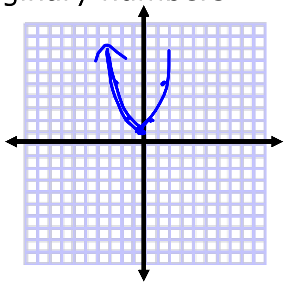
Unit 2.4: Imaginary and Complex Numbers

Objectives:
- Simplify radicals using the imaginary number, i
- Perform operations on complex numbers
- Solve quadratic equations using square roots and imaginary numbers

Consdider the equation: $y = x^2 + 1$



What is the vertex? (0, 1)

What are the x-intercepts? None

Because the graph as NO x-intercepts, we say that the equation has NO solution. To overcome this problem, mathematicians created an expanded system of numbers using the imaginary unit i, defined as $i = \sqrt{-1}$.
Note that $i^2 = -1$. The imaginary unit i can be used to write the square root of any negative number.

Core Concept

The Square Root of a Negative Number

Property	Example
1. If r is a positive real number, then $\sqrt{-r} = i\sqrt{r}$.	$\sqrt{-3} = i\sqrt{3}$
2. By the first property, it follows that $(i\sqrt{r})^2 = -r$.	$(i\sqrt{3})^2 = i^2 \cdot 3 = -3$

Memorize this!!!

$$i = \sqrt{-1}$$
$$i^2 = -1$$
$$i^3 = -i$$
$$i^0 = i^4 = 1$$

- i = imaginary unit

Ex. 1 Simplify.

a. $\sqrt{-25}$

$\sqrt{-1}\ \sqrt{25}$

$i \cdot 5 = 5i$

b. $2\sqrt{-72}$

72
2 36
6 6
3 2 3 2

$i \cdot 2 \cdot 3 \cdot 2\sqrt{2}$

$12i\sqrt{2}$

c. $-5\sqrt{-9}$

$-5 \cdot 3i$

$-15i$

d. $i\sqrt{-27}$

27
9 3
3 3

$i \cdot i \cdot 3\sqrt{3}$

$3i^2\sqrt{3}$

$3(-1)\sqrt{3}$

$-3\sqrt{3}$

Real part Imaginary part

$a + bi$

$16 + 0i$

A **complex number** written in standard form is a number

a + bi where a and b are real numbers. The number a is the real part, and the number bi is the imaginary part.

Real part Imaginary part

$a + bi$

Ex. 2 Add or subtract.

a. $(2+3i)+(5-6i) =$

$7 - 3i$

b. $(3-i)-(5-6i) =$

$3 - i - 5 + 6i =$

$-2 + 5i$

Ex. 3 Multiply.

Remember: $i^2 = -1$

a. $2i \cdot 3i = 6i^2$

$= 6(-1)$

$= -6$

b. $4i(-6+i) = -24i + 4i^2$

$= -24i + 4(-1)$

$= -24i - 4$

$\boxed{-4 - 24i}$

c. $(9-2i)(-4+7i)$

$-36 + 63i + 8i - 14i^2$

$-14(-1)$

$-36 + 71i + 14$

$-22 + 71i$

d. $(5-2i)^2$

$(5-2i)(5-2i)$

$25 - 10i - 10i + 4i^2$ -1

$25 - 20i - 4$

$21 - 20i$

Recall that expressions in simplest form cannot have square roots in the denominator. Because the imaginary unit represents a square root, you must **rationalize any denominator** that contains an imaginary unit. To do this, **multiply the numerator and denominator** by the **complex CONJUGATE of the denominator**.

Helpful Hint

The complex conjugate of a complex number $a + bi$ is $a - bi$. (Lesson 5-5)

Ex. 4 State the complex conjugate for each.

a. $2 + 3i$ b. $-3 - 3i$ c. $2i$

$2-3i$ $-3+3i$ $-2i$

Ex. 5 Simplify.

a. $\dfrac{3+10i}{5i} \cdot \dfrac{-5i}{-5i}$

$= \dfrac{-15i - 50i^2}{-25i^2} = \dfrac{-15i+50}{25} = \dfrac{\cancel{50}^{10} - \cancel{15}^{3}i}{\cancel{25}_{5}}$

b. $\dfrac{2+8i}{4+2i} \cdot \dfrac{4-2i}{4-2i}$

$= \dfrac{8 - 4i + 32i - \cancel{16i^2}^{+16}}{16 - \cancel{4i^2}_{+4}}$

$= \dfrac{24+28i}{20}$

$= \dfrac{6+7i}{5}$

Now you try.....

$\dfrac{4+i}{2-i} \cdot \dfrac{2+i}{2+i} = \dfrac{8+4i+2i-i^2}{4-i^2}$

$= \dfrac{8+6i+1}{4+1} = \dfrac{9+6i}{5}$