Algebra 2

2.5 Solving Quadratic Equations by Completing the Square

Objectives:
- Solve quadratic equations using square roots.
- Solve quadratic equations by Completing the Square.

Solving a Quadratic Equation Using Square Roots

We can solve quadratic equations of the form $ax^2 + c = 0$ by using square roots.

To solve, first isolate x^2 on one side of the equation to obtain $x^2 = d$. Then solve by taking the square root of each side.
Remember: $\pm$ for $\sqrt{\ }$

Ex. 1: Solve using square roots.

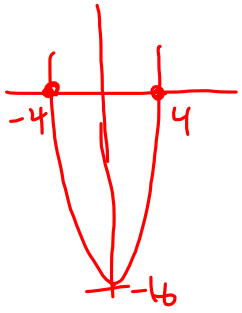
a. $x^2 - 16 = 0$

$y = x^2 - 16$

$+16 \quad +16$

$\sqrt{x^2} = \sqrt{16}$

$x = \pm 4$



b. $3x^2 - 27 = 0$

$+27 \quad +27$

$\frac{3x^2}{3} = \frac{27}{3}$

$\sqrt{x^2} = \sqrt{9}$

$x = \pm 3$

3 or -3

c. $x^2 - 5 = 7$

$+5 \quad +5$

$\sqrt{x^2} = \sqrt{12}$

$x = \pm\sqrt{12}$

$x = \pm 2\sqrt{3}$

12 → 4 · 3 → 2 2

d. $\sqrt{(x+8)^2} = \sqrt{36}$

$x + 8 = \pm 6$

$-8 \quad -8$

$x = -8 \pm 6$ → -2, -14

Solving by Completing the Square

Steps for Solving by Completing the Square

1. If a ≠ 1, divide the equation by a.
2. Move constant term to the right of = sign.
3. Take **½ the middle term**, **square it** and **add it to both sides**.
4. Simplify the right side.
5. Factor left side as a **Perfect Square Trinomial**.
6. **Square root both sides** and solve for x.

Ex. 2: Solve by Completing the Square

Solving $ax^2 + bx + c = 0$ when a = 1

a. $x^2 + 6x - 7 = 0$

$+7 \quad +7$

$x^2+6x+9 = 7+9$

$\frac{1}{2}$ ↓ ↗ 9

$\sqrt{(x+3)^2} = \sqrt{16}$

$x+3 = \pm 4 \rightarrow x = -3 \pm 4$

$-3 \quad -3$

1, −7

b. $x^2 - 10x + 7 = 0$

$-7 \quad -7$

$x^2-10x+25 = -7+25$

$\sqrt{(x-5)^2} = \sqrt{18}$

18 = 6 · 3 = 2 · 3 · 3

$x-5 = \pm\sqrt{18}$

$x-5 = \pm 3\sqrt{2}$

$+5 \quad +5$

$x = 5 \pm 3\sqrt{2}$

c. $x^2 + x + 2 = 32$

$-2 \quad -2$

$x^2+1x+\frac{1}{4} = 30\frac{4}{4}+\frac{1}{4}$

$\sqrt{(x+\frac{1}{2})^2} = \sqrt{\frac{121}{4}}$

$x+\frac{1}{2} = \pm\frac{11}{2}$

$-\frac{1}{2} \quad -\frac{1}{2}$

$x = -\frac{1}{2} \pm \frac{11}{2}$

$\frac{10}{2} = 5$

$\frac{-12}{2} = -6$

Solving $ax^2 + bx + c = 0$ when $a \neq 1$

Ex. 3: Solve by Completing the Square

a. $\frac{4x^2}{4} + \frac{32x}{4} = \frac{-68}{4}$

$x^2 + 8x \underline{\quad +16 \quad} = -17 \underline{\quad +16 \quad}$

$\sqrt{(x+4)^2} = \sqrt{-1}$

$x + 4 = \pm i$

$-4 \quad -4$

$x = -4 \pm i$ or $-4+i$, $-4-i$

b. $\frac{6x^2}{6} + \frac{12x}{6} + \frac{42}{6} = \frac{0}{6}$

$x^2 + 2x + 7 = 0$

$-7 \quad -7$

$x^2 + 2x \underline{\quad +1 \quad} = -7 \underline{\quad +1 \quad}$

$\sqrt{(x+1)^2} = \sqrt{-6}$

$x + 1 = \pm i\sqrt{6}$

$-1 \quad -1$

$x = -1 \pm i\sqrt{6}$