

2.6 Solving Quadratic Equations by factoring

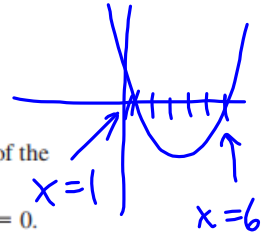
Objective: Find the solutions (zeros) to a quadratic equation.

Core Concept

Zero-Product Property

Words If the product of two expressions is zero, then one or both of the expressions equal zero.

Algebra If A and B are expressions and $AB = 0$, then $A = 0$ or $B = 0$.



Find the value of x .

Ex1) $(x + 4)(x + 7) = 0$

$$\begin{aligned} x+4=0 & \text{ or } x+7=0 \\ x=-4 & \quad x=-7 \end{aligned}$$

Ex2) $2x(3x - 2) = 0$

$$\begin{aligned} 2x=0 & \text{ or } 3x-2=0 \\ x=0 & \quad +2 \quad +2 \\ & \quad \frac{3x}{3} = \frac{2}{3} \\ & \quad x = \frac{2}{3} \end{aligned}$$

We have solved quadratic equations by graphing, square roots and completing the square. Another method we can use is **FACTORING**.

REMEMBER FACTORING:

- Always check for a GCF first $2x + 10 = 2(x + 5)$
- 2 terms- Difference of Squares $25x^2 - 16 = (5x + 4)(5x - 4)$
- 3 terms- Factor directly
 $x^2 - 5x - 6 = (x - 6)(x + 1)$ (yay!)
 $(x - 3)(x + 2)$ (ugh!)
 $2x^2 - x - 3 = (2x - 3)(x + 1)$ (ugh!)

Ex. 3 Solve by factoring:

$$x^2 - 4x = 45$$

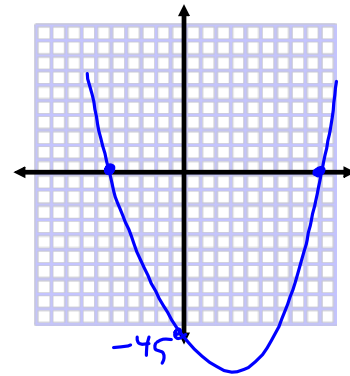
$$\quad -45 \quad -45$$

$$x^2 - 4x - 45 = 0$$

$$(x - 9)(x + 5) = 0$$

$$x = 9 \quad x = -5$$

- Set the equation = 0
- Factor the equation
- Zero-Product Property
- Solve for x



If a real number k is a **zero** of the function $f(x) = ax^2 + bx + c$, then k is an **x-intercept** of the graph of the function, and k is also a **root** of the equation $ax^2 + bx + c = 0$ ∴

zero "root" "x-intercept"

A **zero** of a function, f , is an x -value for which $f(x) = 0$.

Find the ZEROS of the quadratic function:

Ex4) $f(x) = 2x^2 - 11x + 12$

$$0 = 2x^2 - 11x + 12$$

$$0 = (2x - 3)(x - 4)$$

$$2x - 3 = 0 \quad x - 4 = 0$$

$$2x = 3$$

$$x = \frac{3}{2}$$

$$x = 4$$

Ex6) $3x^2 + 5x - 2$

$$-2$$

$$3x^2 + 5x - 2 = 0$$

$$(3x - 1)(x + 2) = 0$$

$$3x - 1 = 0 \quad x + 2 = 0$$

$$3x = 1 \quad x = -2$$

$$x = \frac{1}{3}$$

Ex5) $f(x) = 4x^3 - 36x$

$$0 = 4x^3 - 36x$$

$$0 = 4x(x^2 - 9)$$

$$0 = 4x(x - 3)(x + 3)$$

$$x = 0 \quad x = 3 \quad x = -3$$

Ex7) $2x^2 + 8x = f(x)$

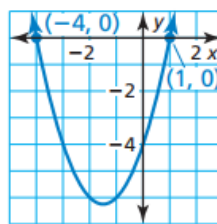
$$2x^2 + 8x = 0$$

$$2x(x + 4) = 0$$

$$x = 0 \quad x = -4$$

Ex8) Use the x-intercepts to write a quadratic function in standard form.

$$\begin{array}{r} x = -4 \\ +4 \quad +4 \\ x + 4 = 0 \end{array} \quad \begin{array}{r} x = 1 \\ -1 \quad -1 \\ x - 1 = 0 \end{array}$$



$$(x+4)(x-1) = 0$$

$$x^2 - 1x + 4x - 4 = 0 \rightarrow x^2 + 3x - 4 = f(x)$$

Ex9) Write a quadratic function in standard form with zeros $1/5$ and -5 .

$$\begin{array}{r} x = \frac{1}{5} \\ -\frac{1}{5} \quad -\frac{1}{5} \\ 5 \left(x - \frac{1}{5} \right) = 0.5 \\ 5x - 1 = 0 \end{array} \quad \begin{array}{r} x = -5 \\ +5 \quad +5 \\ x + 5 = 0 \\ x + 5 = 0 \end{array}$$

$$(5x-1)(x+5) = 0$$

$$5x^2 + 25x - x - 5 = 0$$

$$5x^2 + 24x - 5 = f(x)$$