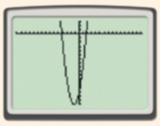
2.7 The Quadratic Formula

Learning Targets: Solve quadratic equations using the Quadratic Formula.
Classify roots using the discriminant.

5 ways to solve a quadratic:

1. Graphing ✓
2. Factoring ✓
3. Square Roots ✓
4. Completing the Square ✓
5. Quadratic Formula

Method	When to Use	Examples
Graphing	Only approximate solutions or the number of real solutions is needed.	$2x^2 + 5x - 14 = 0$  $x \approx -4.2$ or $x \approx 1.7$
Factoring	$c = 0$ or the expression is easily factorable.	$x^2 + 4x + 3 = 0$ $(x + 3)(x + 1) = 0$ $x = -3$ or $x = -1$
Square roots	The variable side of the equation is a perfect square.	$(x - 5)^2 = 24$ $\sqrt{(x-5)^2} = \pm\sqrt{24}$ $x - 5 = \pm 2\sqrt{6}$ $x = 5 \pm 2\sqrt{6}$
Completing the square	$a = 1$ and b is an even number.	$x^2 + 6x = 10$ $x^2 + 6x + \square = 10 + \square$ $x^2 + 6x + \left(\frac{6}{2}\right)^2 = 10 + \left(\frac{6}{2}\right)^2$ $(x + 3)^2 = 19$ $x = -3 \pm \sqrt{19}$
Quadratic Formula	Numbers are large or complicated, and the expression does not factor easily.	$5x^2 - 7x - 8 = 0$ $x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(5)(-8)}}{2(5)}$ $x = \frac{7 \pm \sqrt{209}}{10}$

Core Concept

The Quadratic Formula

Let a, b, and c be real numbers such that $a \neq 0$. The solutions of the quadratic equation $ax^2 + bx + c = 0$ are $\boxed{x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$.

IF

Then

Example 1:

Find the zeros of $x^2 + 3x = 5$ using the Quadratic Formula.

$$x^2 + 3x - 5 = 0$$

$a = 1 \quad b = 3 \quad c = -5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-3 \pm \sqrt{3^2 - 4(1)(-5)}}{2(1)}$$

$$= \frac{-3 \pm \sqrt{9 + 20}}{2}$$

$$= \frac{-3 \pm \sqrt{29}}{2}$$

Example 2:

Solve $25x^2 - 8x = 12x - 4$ using the Quadratic Formula.

$-12x \quad -12x + 4$

$+4$

$25x^2 - 20x + 4 = 0$

$a = 25 \quad b = -20 \quad c = 4$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{20 \pm \sqrt{400 - 400}}{50}$

$\frac{20 \pm \sqrt{0}}{50} = \frac{20 \pm \cancel{0}}{50}$

$= \frac{20}{50} = \frac{2}{5}$

Example 3:

Solve $-x^2 + 4x = 13$ using the Quadratic Formula.

$-13 \quad -13$

$-x^2 + 4x - 13 = 0$

$a = -1 \quad b = 4 \quad c = -13$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-4 \pm \sqrt{16 - 52}}{-2}$

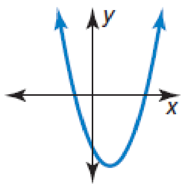
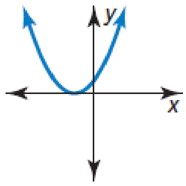
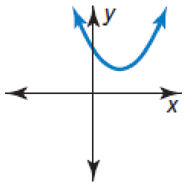
$= \frac{-4 \pm \sqrt{-36}}{-2} = \frac{\overset{2}{\cancel{-4}} \pm \overset{-3}{\cancel{6}}i}{\underset{1}{\cancel{-2}}}$

$= \frac{2 \pm 3i}{1} = 2 \pm 3i$

Core Concept

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Analyzing the Discriminant of $ax^2 + bx + c = 0$

Value of discriminant	$b^2 - 4ac > 0$	$b^2 - 4ac = 0$	$b^2 - 4ac < 0$
Number and type of solutions	Two real solutions	One real solution	Two imaginary solutions
Graph of $y = ax^2 + bx + c$	 Two x-intercepts	 One x-intercept	 No x-intercept

Perfect Square 2 Real, Rational
Not Perfect Square 2 Real, Irrational

Example 4:

Find the discriminant of the quadratic equation and describe the number and type of solutions of the equation.

a. $x^2 - 6x + 10 = 0$

$a=1 \quad b=-6 \quad c=10$

b^2-4ac

$36-40=-4<0$

2 imaginary

b. $x^2 - 6x + 9 = 0$

$a=1 \quad b=-6$
$c=9$

b^2-4ac

$36-36=0$

1 real soln.

c. $x^2 - 6x + 8 = 0$

$a=1 \quad b=-6 \quad c=8$

$b^2-4ac=36-32=4$

$4>0$

2 real solns.

Example 5:

Find a possible pair of integer values for *a* and *c* so that the equation $ax^2 - 4x + c = 0$ has one real solution. Then write the equation.

$$b^2 - 4ac = 0$$

$$16 - 4ac = 0$$

$$\frac{16}{4} = \frac{4ac}{4}$$

$$4 = ac$$

2 & 2
4 & 1
1 & 4
-1 & -4
-4 & -1
-2 & -2

any of these pairs

Concept Summary

Methods for Solving Quadratic Equations

Method	When to Use
Graphing	Use when approximate solutions are adequate.
Using square roots	Use when solving an equation that can be written in the form $u^2 = d$, where u is an algebraic expression.
Factoring	Use when a quadratic equation can be factored easily.
Completing the square	Can be used for *any* quadratic equation $ax^2 + bx + c = 0$ but is simplest to apply when $a = 1$ and b is an even number.
Quadratic Formula	Can be used for *any* quadratic equation.