

2.8 : Quadratic Applications

Objectives:

- identify the following key characteristics of functions from graphs, tables and equations: domain, range, end behavior, x- and y-intercepts, local maxima and minima values, symmetries, intervals of increasing and decreasing.
- use quadratics to solve real world problems.

$$h(t) = -16t^2 + v_0t + h_0$$

t: time

v_0 : initial velocity

h_0 : initial height

Ex. 1 After the semester is over, Tim and Herman, discover that the math department at their university has changed textbooks and so, the bookstore won't buy back their math books. Tim and Herman decide to get creative. They go to the roof of a twelve-story building and look over the edge to the reflecting pool 160 feet below. Tim drops his book over the edge at the same instant that Herman chucks his book straight down at 48 feet per second. These two equations model the height of the books (in feet) at the time, t seconds, after each student releases the book.

Write the equations:

Tim: $s(t) = -16t^2 + \overset{\text{drop}}{\cancel{0}t} + 160$

Herman: $s(t) = -16t^2 - 48t + 160$

When does each student's book hit the water?

$$0 = \frac{-16t^2}{-16} - \frac{48t}{-16} + \frac{160}{-16}$$

$$0 = t^2 + 3t - 10$$

$$0 = (t + 5)(t - 2)$$

$$\cancel{t = -5} \quad t = 2 \text{ sec.}$$

$$h(t) = -16t^2 + v_0t + h_0$$

$$s(t) = -16t^2 + 160$$

$$0 = -16t^2 + 160$$

$$\frac{16t^2}{16} = \frac{160}{16}$$

$$\sqrt{t^2} = \sqrt{10}$$

$$t = \pm 3.16$$

We want

3.16 sec

positive only

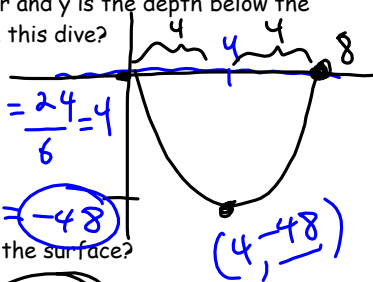
Ex. 2 Suppose a penguin is swimming on the surface of the water and makes a dive in a parabolic path described by $y=3x(x-8)$ where x is the horizontal distance in feet the penguin swims underwater and y is the depth below the surface. What is the maximum depth under the surface that the penguin swims in this dive?

$$y = 3x(x-8)$$

$$y = 3x^2 - 24x$$

$$x = \frac{-b}{2a} = \frac{-(-24)}{2(3)} = \frac{24}{6} = 4$$

$$y = 3(4)^2 - 24(4) = 48 - 96 = -48$$



How far from the initial diving point is the spot where the penguin comes back to the surface?

$$0 = 3x(x-8)$$

$$3x = 0 \quad x - 8 = 0$$

$$x = 0 \quad x = 8$$

What is the domain and range? Describe what it means in context.

D: $[0, 8]$ The penguin traveled 8ft. horizontally

R $[-48, 0]$ The penguin dove 48 ft. below the surface

Ex. 3 Sally is standing in her yard and throws a helium-filled balloon downward toward the ground. Suppose that y is the height above the ground (in feet) at the time x seconds after she releases the balloon. We model this with $y=5x^2-10x+5$. For what interval of time is the balloon moving downward toward the ground? For what interval of time does the equation say that the balloon is rising? How far above the ground is the balloon 4 seconds after she released it?

$$y = 5x^2 - 10x + 5$$



$$x = \frac{-b}{2a} = \frac{-(-10)}{2(5)} = \frac{10}{10} = 1$$

balloon downward
 $(0, 1)$

upward
 $(1, \infty)$

falling changes to rising

How far above at 4 sec?

4 in for x

$$y = 5(4)^2 - 10(4) + 5$$

$$= 5(16) - 40 + 5$$

$$= 80 - 40 + 5 = 45 \text{ feet above ground}$$