Algebra 2

3.2 Dividing Polynomials Using Long Division

<u>Objective</u>: Use Long Division and Synthetic Division to divide polynomials.

Ex. 1: $\frac{156}{4}$

$$4\overline{)156} = 39$$
-12
36
-36
0

$$\text{divisor}\overline{)\text{dividend}} \quad \text{quotient}$$

- When dividing polynomials, you use a process similar to long division

Ex. 2: $(6r^2s^2 + 3rs^2 - 9r^2s) \div (3rs)$

Quotient: $2rs + 1s - 3r$

$$3rs\overline{)6r^2s^2 + 3rs^2 - 9r^2s}$$
$-6r^2s^2$
$0 + 3rs^2$
$-3rs^2$
$0 - 9r^2s$
$+(+9r^2s)$
0

Ex. 2: $(c^2 - c - 30) \div (c - 6)$

Quotient: $c + 5$

$$c - 6\overline{)c^2 - c - 30}$$
$-c^2 + 6c$
$5c - 30$
$-5c + 30$
0

$$\boxed{\frac{5\cancel{c}}{\cancel{c}} = 5}$$

check
$(c-6)(c+5)$

Ex. 3: $(6n^2 - 3n + 4) \div (2n - 3)$

Quotient: $3n + 3 + \frac{13}{2n-3}$

$$2n - 3\overline{)6n^2 - 3n + 4}$$
$-6n^2 + 9n$
$6n + 4$
$-6n + 9$
13

- If you get a remainder, write your answer in this form:

$$\text{Quotient} + \frac{\text{Remainder}}{\text{Divisor}}$$

There is a shortcut for dividing polynomials by binomials of the form $x - k$. This shortcut is called synthetic division.

Ex. 4: $(6x^3 - 19x^2 + x + 6) \div (x - 3)$

$x - 3 = 0$
$x = 3$

$$
\begin{array}{r|rrrr}
3 & 6x^3 & -19 & 1 & 6 \\
 & \downarrow & 18 & -3 & -6 \\
\hline
 & 6x^2 & -1x & -2 & \boxed{0}
\end{array}
$$

$6x^2 - 1x - 2$

Set the divisor = 0 and solve. Put that number in the box.

Put the poly. in order from highest to lowest degree

Write the coefficients across the top (if you are missing a term, fill in with 0)

Bring down the 1st number

Mult. that number by the # in the box

Write the product under the second #, add

Repeat the process

Box off the last number, this is the remainder

Working backwards, starting with the constant, put the variables on the numbers. This is your quotient.

Write the answer as:

$$\text{Quotient} + \frac{\text{Remainder}}{\text{Divisor}}$$

ex 5 $(-5x^5 - 21x^4 - 3x^3 + 4x^2 - 2x + 2) \div (x+4)$

$x + 4 = 0$
$x = -4$

$$
\begin{array}{r|rrrrrr}
-4 & -5x^5 & -21 & -3 & 4 & -2 & 2 \\
 & \downarrow & 20 & 4 & -4 & 0 & 8 \\
\hline
 & -5x^4 & -1x^3 & +1x^2 & +0x & -2 & \boxed{10}
\end{array}
$$

$$-5x^4 - x^3 + x^2 - 2 + \frac{10}{x+4}$$

ex 6 $(x^4 - 2x^3 + x - 1) \div (x+1)$

$x+1=0$

$x=-1$

$$
\begin{array}{r|rrrrr}
-1 & 1 & -2 & 0 & 1 & -1 \\
 & \downarrow & -1 & 3 & -3 & 2 \\
\hline
 & 1 & -3 & 3 & -2 & \boxed{1}
\end{array}
$$

$$x^3 - 3x^2 + 3x - 2 + \frac{1}{x+1}$$

Ex. 7: $(4x^4 - 5x^2 + 2x + 4) \div (2x - 1)$

$2x-1=0$

$2x=1$

$x=\frac{1}{2}$

$$
\begin{array}{r|rrrrr}
\frac{1}{2} & 4 & 0 & -5 & 2 & 4 \\
 & \downarrow & 2 & 1 & -2 & 0 \\
\hline
 & 4 & 2 & -4 & 0 & \boxed{4}
\end{array}
$$

$$4x^3 + 2x^2 - 4x + 0 + \frac{4}{2x-1}$$

Core Concept

The Remainder Theorem

If a polynomial $f(x)$ is divided by $x - k$, then the remainder is $r = f(k)$.

Use synthetic division to evaluate the polynomial for the given value.

Ex. 8: $P(x) = 2x^3 + 5x^2 - x + 7$ for $x = 2$

$$\begin{array}{r|rrrr} 2 & 2 & 5 & -1 & 7 \\ & \downarrow & 4 & 18 & 34 \\ \hline & 2 & 9 & 17 & \boxed{41} \end{array}$$

$P(2) = 41$

Ex. 9: $f(x) = 4x^2 - 10x - 21$; $x = 5$

$$\begin{array}{r|rrr} 5 & 4 & -10 & -21 \\ & \downarrow & 20 & 50 \\ \hline & 4 & 10 & \boxed{29} \end{array}$$

$f(5) = 29$

Ex. 5: $(-5x^5 - 21x^4 - 3x^3 + 4x^2 - 2x + 2) \div (x + 4)$

Ex. 6: $(x^4 - 2x^3 + x - 1) \div (x + 1)$