

Day 4 Factor Polynomials

Objective: Use the Factor Theorem to determine whether linear binomials are factors of polynomials.

Core Concept

The Factor Theorem

A polynomial $f(x)$ has a factor $x - k$ if and only if $f(k) = 0$.

Example 1:

Determine whether $x - 2$ is a factor of $f(x) = x^2 + 2x - 4$.

$$\begin{array}{r|rrr} 2 & 1 & 2 & -4 \\ & \downarrow & 2 & 8 \\ \hline & 1 & 4 & 4 \end{array} \rightarrow \text{not } 0 \text{ so } x-2 \text{ is not a factor.}$$

Example 2:

Determine whether $x + 5$ is a factor of $f(x) = 3x^4 + 15x^3 - x^2 + 25$.

$$\begin{array}{r}
 \begin{array}{cccccc}
 \xrightarrow{\quad} & & & & \uparrow & \\
 & & & & \text{no } x & \\
 -5 \Big| & 3 & 15 & -1 & 0 & 25 \\
 & \downarrow & -15 & 0 & 5 & -25 \\
 \hline
 & 3 & 0 & -1 & 5 & \underline{0}
 \end{array}
 \end{array}
 \rightarrow \begin{array}{l}
 \text{'is } 0, \text{ so} \\
 x+5 \text{ is a factor}
 \end{array}$$

Example 3:

Show that $x + 3$ is a factor of $f(x) = x^4 + 3x^3 - x - 3$. Then factor $f(x)$ completely.

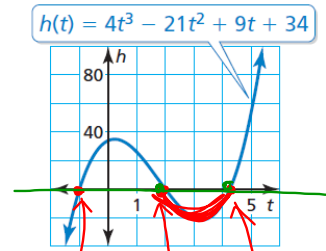
$$\begin{array}{r}
 \begin{array}{cccccc}
 & & & & \text{no } x^2 & \\
 & & & & \swarrow & \\
 -3 \Big| & x^4 & 3 & 0 & -1 & -3 \\
 & \downarrow & -3 & 0 & 0 & 3 \\
 \hline
 & x^3 & 0 & 0 & -1 & \underline{0}
 \end{array}
 \end{array}
 \rightarrow \begin{array}{l}
 \text{'is } 0 \text{ so } x+3 \\
 \text{'is a factor}
 \end{array}$$

$$x^3 - 1 = (x - 1)(x^2 + x + 1)$$

$$(x+3)(x-1)(x^2+x+1)$$

Example 4:

During the first 5 seconds of a roller coaster ride, the function $h(t) = 4t^3 - 21t^2 + 9t + 34$ represents the height h (in feet) of the roller coaster after t seconds. How long is the roller coaster at or below ground level in the first 5 seconds?



$$\begin{array}{r}
 x = 2 \\
 \hline
 2 \overline{) 4x^3 - 21x^2 + 9x + 34} \\
 \underline{8x^2 - 26x + 34} \\
 4x^2 - 13x - 17 \quad \boxed{0} \\
 4x^2 - 13x - 17 = 0
 \end{array}$$

$$\begin{array}{r}
 -1 \overline{) 4x^2 - 13x - 17} \\
 \underline{4x^2 - 4x + 17} \\
 4x - 17 \quad \boxed{0} \\
 4x - 17 = 0 \rightarrow x = \frac{17}{4}
 \end{array}$$

$$\begin{aligned}
 &A=4 \quad B=-13 \quad C=-17 \\
 &x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} = \frac{13 \pm \sqrt{169 + 272}}{8} \\
 &= \frac{13 \pm \sqrt{441}}{8} = \frac{13 \pm 21}{8} \\
 &\quad \swarrow \quad \searrow \\
 &\frac{34}{8} = \frac{17}{4} \quad \frac{-8}{8} = -1
 \end{aligned}$$

$[2, 4\frac{1}{4}]$ so $2\frac{1}{4}$ sec.