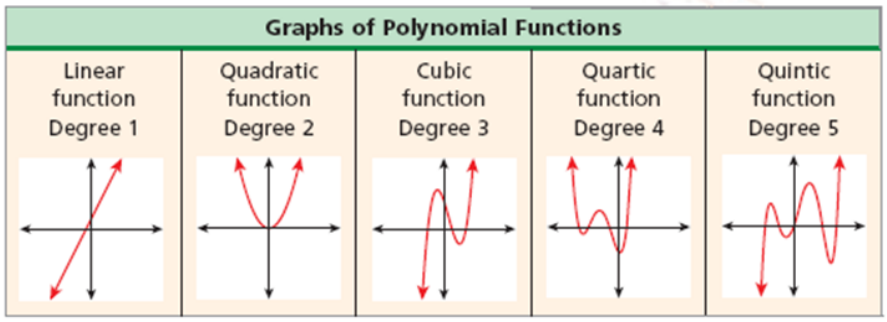
3.5 Investigating Graphs of Polynomial Functions

- Use properties of end behavior to analyze, describe, and graph polynomial functions

The graphs of polynomial functions are classified by the degree of the polynomial. Each graph, based on the degree, has a distinctive shape and characteristics.



Maximum # of x-intercepts = degree	1	2	3	4	5
Maximum # of turning points (changes in direction) = degree -1	0	1	2	3	4

End behavior is a description of the values of the function as x approaches infinity ($x \to +\infty$) or negative infinity ($x \to -\infty$). The **degree** and **leading coefficient** of a polynomial function determine its end behavior.

Polynomial End Behavior

$P(x)$ has...	Odd Degree	Even Degree
Leading coefficient $a > 0$	As $x \to +\infty$, $P(x) \to +\infty$ As $x \to -\infty$, $P(x) \to -\infty$	As $x \to -\infty$, $P(x) \to +\infty$ As $x \to +\infty$, $P(x) \to +\infty$
Leading coefficient $a < 0$	As $x \to -\infty$, $P(x) \to +\infty$ As $x \to +\infty$, $P(x) \to -\infty$	As $x \to -\infty$, $P(x) \to -\infty$ As $x \to +\infty$, $P(x) \to -\infty$

a positive

a negative

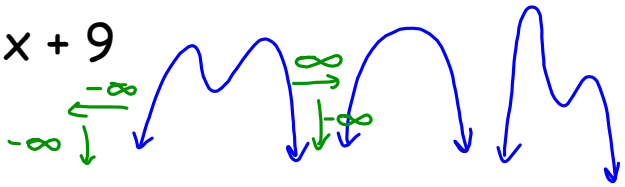
Identify the leading coefficient, degree, and end behavior.

Ex 1) $Q(x) = -x^4 + 6x^3 - x + 9$

leading coefficient: -1

degree: 4

end behavior: $x \to -\infty, Q(x) \to -\infty$ & $x \to \infty, Q(x) \to -\infty$

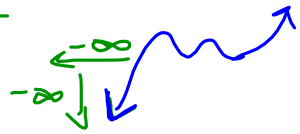


Ex 2) $P(x) = 2x^5 + 6x^4 - x + 4$

leading coefficient: 2

degree: 5

end behavior:

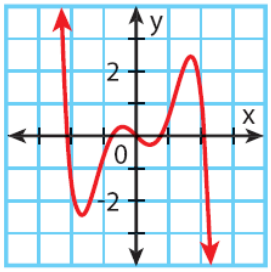


as $x \to -\infty$, $P(x) \to -\infty$

as $x \to \infty$, $P(x) \to \infty$

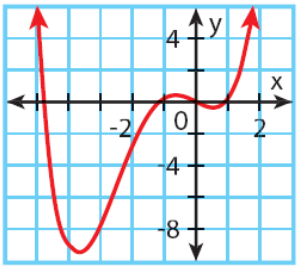
Identify whether the function graphed has an odd or even degree and a positive or negative leading coefficient

Ex 3)



odd degree
(opposite directions)
lead coeff. is negative

Ex 4)



even degree
(goes back where it came from)
lead coeff. is positive

Ex 5) Graph a function based on these characteristics.

Leading Coefficient: Negative

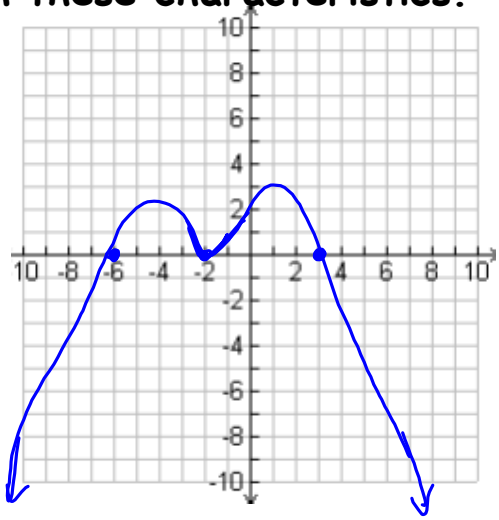
Degree: Even →

Zeros: -6, -2, -2, 3

roots or x-intercepts

We say that -2 has a multiplicity of 2.

We could also say that -2 is a double root.



Ex 6) Graph the function. $f(x) = x^3 + 4x^2 + x - 6$

zeros: -3, -2, 1

Degree: 3

Leading Coefficient: +1

End Behavior:

as $x \to -\infty$, $f(x) \to -\infty$

& as $x \to \infty$, $f(x) \to \infty$

x	y
0	-6
2	12