Algebra 2

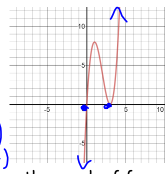
3.6 Finding Real Roots of Polynomial Equations

Objectives:
- Identify the multiplicity of roots.
- Use Rational Root Theorem to solve equations.

When an answer appears more than once, it is called a repeated solution. The number of times a solution repeats is the multiplicity of the root.

Solve each polynomial by FACTORING and state the multiplicity of each root.

Ex. 1: $2x^3 - 12x^2 + 18x = 0$



$2x(x^2-6x+9)=0$

$2x(x-3)(x-3)=0$

$2x=0 \quad x-3=0 \quad x-3=0$

$x=0$ (single root) $\quad x=3$ (multiplicity 2 or double root)

- When a factor $(x - k)$ of $f(x)$ is raised to an even power, the graph of f "bounces" or touches the x-axis (but does not cross the x-axis) at x = k.
- When a factor $x - k$ of $f(x)$ is raised to an odd power, the graph of f crosses the x-axis at x = k.

Ex. 2: $5x^3 - 40x^2 + 80x = 0$

$5x(x^2-8x+16)=0$

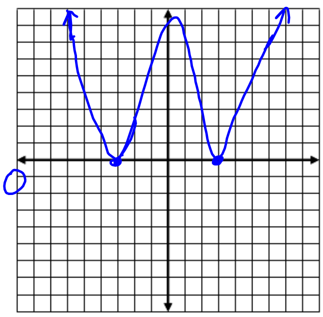
$5x(x-4)(x-4)=0$

$x=0 \quad x=4 \quad x=4$

single root, multiplicity 1 $\quad$ double root, multiplicity 2

Find the zeros and sketch the graph of each polynomial function.

Ex. 3: $f(x) = x^4 - 18x^2 + 81$



$(x^2-9)(x^2-9)$

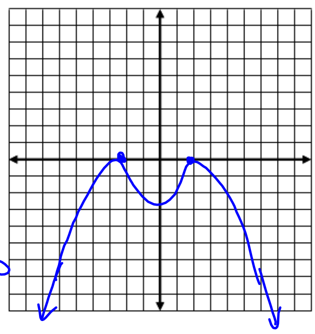
$(x+3)(x-3)(x+3)(x-3)=0$

$x=-3 \quad x=3$

both double roots

Ex. 4: $\cancel{f(x)=}\ -2x^4 + 16x^2 - 32 = 0$

$\frac{-2x^4}{-2} + \frac{16x^2}{-2} - \frac{32}{-2} = \frac{0}{-2}$



$x^4 - 8x^2 + 16$

$(x^2-4)(x^2-4)$

$(x+2)(x-2)(x+2)(x-2)=0$

$x=-2 \quad x=2$

each is multiplicity 2

Not all polynomials are factorable, so we have the RATIONAL ROOT THEOREM to help find all possible rational roots.

Core Concept

The Rational Root Theorem

If $f(x) = a_n x^n + \cdots + a_1 x + a_0$ has *integer* coefficients, then every rational solution of $f(x) = 0$ has the following form:

$$\frac{p}{q} = \frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}$$

Ex. 5: List all possible roots for $3x^3 - 4x^2 - 17x + 6$.

$\pm 1, \pm 3$
qs

$\pm 1, \pm 2, \pm 3, \pm 6$
ps

$\frac{p}{q}$: $\pm 1, \pm \frac{1}{3}, \pm 2, \pm \frac{2}{3}, \pm 3, \cancel{\pm \frac{3}{3}}, \pm 6, \cancel{\pm \frac{6}{3}}$

Finding All Roots of a Polynomial

Step 1: Use the Rational Root Theorem (p's and q's) to identify rational roots.
Step 2: Test the possible real roots using Synthetic Division.
Step 3: Use Synthetic Division and the depressed polynomial to find real roots, until the polynomial is reduced to a quadratic.
Step 4: Solve the quadratic by factoring, square roots, completing the square or the quadratic formula to find the remaining roots.

Ex. 6: List all possible rational roots, then FIND all roots.

$f(x) = x^3 + 3x^2 - 25x + 21$

qs: 1

ps: 1, 3, 7, 21

$\frac{p}{q}$: $\pm 1, \pm 3, \pm 7, \pm 21$

1	1	3	-25	21
		1	4	-21
	1	4	-21	0

$x = 1$

$x^2 + 4x - 21 = 0$
$(x - 3)(x + 7) = 0$

$x = 3$ $x = -7$

List all possible rational roots, then FIND all roots.

Ex. 7: $f(x) = 6x^4 + x^3 - 41x^2 - 44x - 12$

1, 2, 3, 6

1, 2, 3, 4, 6, 12

$\frac{p}{q}$: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm \frac{2}{3}, \pm \frac{1}{3}, \pm \frac{3}{2}, \pm \frac{1}{2}, \pm \frac{4}{3}$

-2	$6x^4$	1	-41	-44	-12
		-12	22	38	12
3	$6x^3$	-11	-19	-6	0
		18	21	6	
	$6x^2$	7	2	0	

$6x^2 + 7x + 2 = 0$

$(3x + 2)(2x + 1) = 0$

$3x + 2 = 0$ $\quad x = -\frac{2}{3}$

$2x + 1 = 0$ $\quad x = -\frac{1}{2}$

$-2, 3, -\frac{2}{3}, -\frac{1}{2}$

List all possible rational roots, then FIND all roots.

Ex. 8: $f(x) = x^4 - 3x^3 - 26x^2 + 78x + 40$

1

1, 2, 4, 5, 8, 10, 20, 40

$\frac{p}{q}$: $\pm 1, \pm 2, \pm 4, \pm 5, \pm 8, \pm 10, \pm 20, \pm 40$

-5	$1x^4$	-3	-26	78	40
		-5	40	-70	-40
4	$1x^3$	-8	14	8	0
		4	-16	-8	
	$1x^2$	-4	-2	0	

$x^2 - 4x - 2 = 0$

$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

$= \frac{4 \pm \sqrt{16 + 8}}{2} = \frac{4 \pm \sqrt{24}}{2}$

$= \frac{4 \pm 2\sqrt{6}}{2} = 2 \pm \sqrt{6}$

$4, -5, 2 \pm \sqrt{6}$

Assignment: Worksheet 3.6

Q: Why is the Rational Roots Theorem so polite?

A: [redacted]