Bell Ringer

Write polynomial function given the zeros.

1) $x = \left\{-4, \frac{2}{3}, 1\right\}$

$x = -4$ | $x = \frac{2}{3}$ | $x = 1$

$x + 4 = 0$ | $3x = 2$ | $x - 1 = 0$

$3x - 2 = 0$

$$(x+4)(3x-2)(x-1) = 0$$
$$(3x^2 + 10x - 8)(x-1) = 0$$
$$3x^3 + 7x^2 - 18x + 8 = 0$$
$$f(x) = 3x^3 + 7x^2 - 18x + 8$$

3.8 Creating Polynomial Functions Using The Fundamental Theorem of Algebra

Learning Target:
Write polynomial function given the roots using the fundamental theorem of algebra and the rules of conjugates.

Ex 1: Write the simplest polynomial function with the given zeros.

$x = -4,\ x = -1,\ x = \frac{1}{3}$

$+4 \quad +4 \quad +1 \quad +1 \quad -\frac{1}{3} \quad -\frac{1}{3}$

$x + 4 = 0 \quad x + 1 = 0 \quad 3\left(x - \frac{1}{3}\right) = 0 \cdot 3$

$3x - 1 = 0$

$(x+4)(x+1)(3x-1) = 0$

$(x^2 + 5x + 4)(3x - 1) = 0$

$3x^3 - x^2 + 15x^2 - 5x + 12x - 4 = 0$

$3x^3 + 14x^2 + 7x - 4 = 0$

$f(x)$

Notice that the degree of the function is the same as the number of zeros. This is true for all polynomial functions. However, all of the zeros are not necessarily real zeros. Polynomials functions, like quadratic functions, may have complex zeros that are not real numbers.

Core Concept

The Fundamental Theorem of Algebra

Theorem If $f(x)$ is a polynomial of degree n where $n > 0$, then the equation $f(x) = 0$ has at least one solution in the set of complex numbers.

Corollary If $f(x)$ is a polynomial of degree n where $n > 0$, then the equation $f(x) = 0$ has exactly n solutions provided each solution repeated twice is counted as two solutions, each solution repeated three times is counted as three solutions, and so on.

Ex 2: Write the simplest polynomial function with the given roots.

$x = -1, 5, \sqrt{3}, -\sqrt{3}$

Recall: Irrational roots come in conjugate pairs: $a+\sqrt{b},\ a-\sqrt{b}$

$x=-1 \quad x=5 \quad x=\sqrt{3} \quad x=-\sqrt{3}$

$x+1=0 \quad x-5=0 \quad x-\sqrt{3}=0 \quad x+\sqrt{3}=0$

$(x+1)(x-5)(x-\sqrt{3})(x+\sqrt{3})$

$(x^2-5x+x-5)(x^2-3)$

$(x^2-4x-5)(x^2-3)$

$x^4-3x^2-4x^3+12x-5x^2+15$

$f(x)= x^4-4x^3-8x^2+12x+15$

Ex 3: Write the simplest polynomial function with the given roots.

Recall: Complex roots come in conjugate pairs: $a+bi,\ a-bi$

Recall: Irrational roots come in conjugate pairs: $a+\sqrt{b}$ and $a-\sqrt{b}$.

a) $x=-3,\ x=2+i \quad x=2-i$

$+3 \quad +3 \quad -2\ -i \quad -2\ -i \quad -2\ +i \quad -2\ +i$

$(x+3)(x-2-i)(x-2+i)$

$-(-1)=+1$

$(x+3)(x^2-2x+\cancel{ix}-2x-\cancel{ix}+4-\cancel{2i}-i^2+\cancel{2i})$

$(x+3)(x^2-4x+5)$

$x^3-4x^2+5x+3x^2-12x+15$

$x^3-x^2-7x+15=p(x)$

b) $x=i,\ x=1-\sqrt{2}$

$x=i \quad x=-i \quad x=1+\sqrt{2} \quad x=1-\sqrt{2}$

$(x-i)(x+i)(x-1-\sqrt{2})(x-1+\sqrt{2})$

$(x^2-i^2)(x^2-x+\cancel{x\sqrt{2}}-x+1-\cancel{\sqrt{2}}\ -\cancel{x\sqrt{2}}+\cancel{\sqrt{2}}-2)$

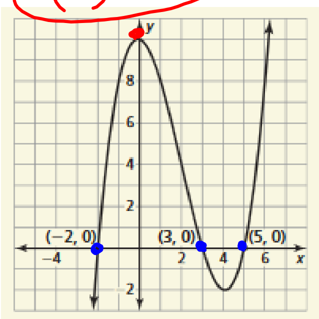
$(x^2+1)(x^2-2x-1)$

$x^4-2x^3-x^2+x^2-2x-1$

$x^4-2x^3-2x-1=0$

Ex 4: Write the polynomial function for a specific y-intercept

(0, 10)



$(x+2)(x-3)(x-5)$

$(x^2-x-6)(x-5)$

$x^3-5x^2-x^2+5x-6x+30$

$$y = \frac{x^3-6x^2-x+30}{3}$$

or

$$y = \frac{1}{3}x^3-2x^2-\frac{1}{3}x+10$$