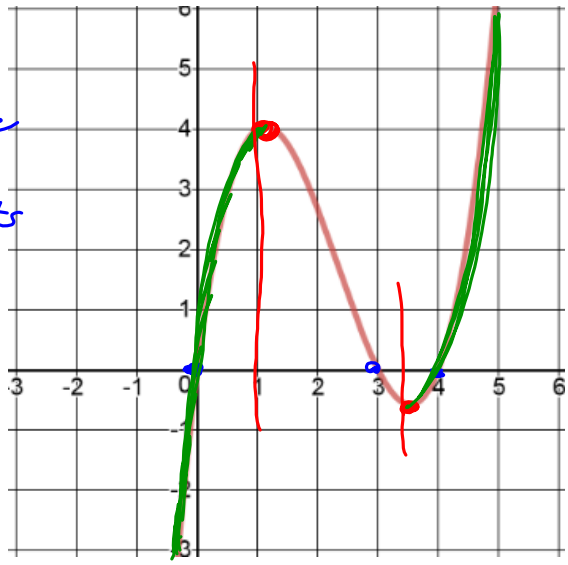
3.9 Graphing polynomial functions

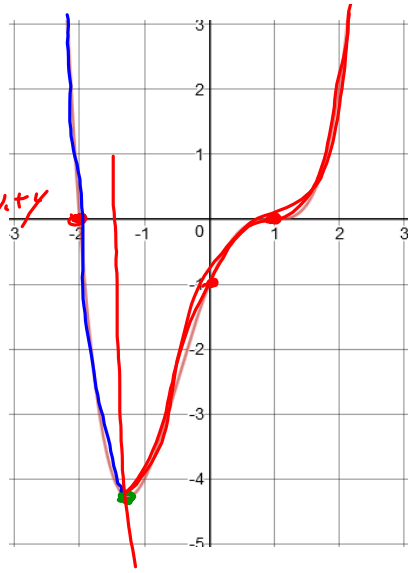
Objective: Given a polynomial function,
identify:
possible rational zeros
all zeros, real or complex
y-intercept
end behavior
domain
approximate:
intervals where increasing or decreasing
maximum or minimum values
range
graph the function

Ex. 1 Based on the graph, identify...



whether odd or even degree: odd
the sign of the lead coefficient: positive
zeros (with multiplicities): 0, 3, 4 single roots
y-intercept: 0
end behavior: $x \to \infty$ $f(x) \to \infty$
$x \to -\infty$ $f(x) \to -\infty$
domain: $(-\infty, \infty)$ ← xs
range: $(-\infty, \infty)$ ← ys
also approximate the...
local maximum: 4
local minimum: $-\frac{1}{2}$
interval(s) where increasing: $(-\infty, 1)$ or $(3.5, \infty)$
interval(s) where decreasing: $(1, 3.5)$

Ex. 2 Based on the graph, identify...



whether odd or even degree: even
sign of the lead coefficient: positive
zeros (with multiplicities): -2, 1 both are odd multiplicity
y-intercept: -1
end behavior: $x \to \infty$ $f(x) \to \infty$
$x \to -\infty$ $f(x) \to \infty$
domain: $(-\infty, \infty)$

also approximate the...
minimum value: -4.25
range: $[-4.25, \infty)$
interval(s) where increasing: $(-1.5, \infty)$
interval(s) where decreasing: $(-\infty, -1.5)$

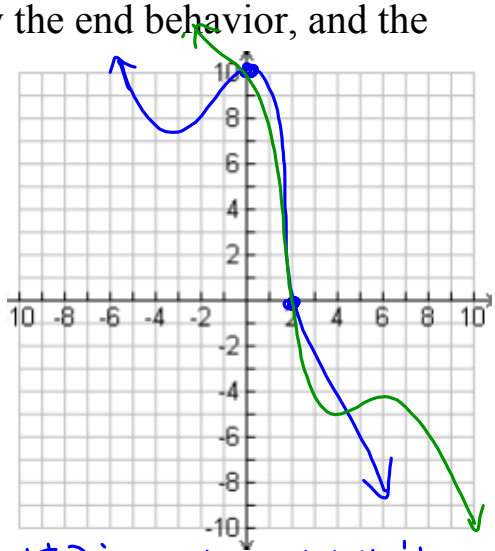
Ex. 3 Given that x=2 is a zero of $y=-x^3 - x + 10$, find the other zeros (with multiplicities), the y-intercept, identify the end behavior, and the domain. Finally, graph the function.

$(-\infty, \infty)$

$$\begin{array}{r|cccc} 2 & -1 & 0 & -1 & 10 \\ & & -2 & -4 & -10 \\ \hline & -1 & -2 & -5 & 0 \end{array}$$

$-x^2 - 2x - 5$

$$x = \frac{2 \pm \sqrt{4-20}}{-2} = \frac{2 \pm \sqrt{-16}}{-2} = \frac{2 \pm 4i}{-2} = -1 \pm 2i$$ single multiplicity



$x \to \infty$ $y \to -\infty$
$x \to -\infty$ $y \to \infty$

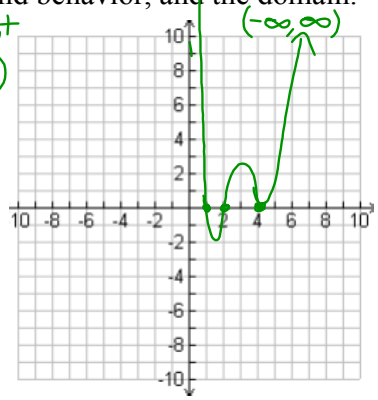
Ex. 4 For $y=x^4 -11x^3 + 42x^2 -64x + 32$, identify the possible rational zeros, then, given that 1 and 2 are zeros, identify the other zeros (with multiplicities), the y-intercept, identify the end behavior, and the domain. Finally, graph the function.

p: 1, 2, 4, 8, 16, 32

q: 1

$\pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm 32$

y-intercept (0, 32)



$x \to -\infty \quad y \to \infty$

$x \to \infty \quad y \to \infty$

1 | 1 −11 42 −64 32

m: 1 | 1 −10 32 −32

2 | 1 −10 32 −32 0

m: 1 | 2 −16 32

| 1 −8 16 0

$x^2 - 8x + 16 = 0$

$(x-4)(x-4) = 0$

$x = 4$ with m: 2