Algebra 2

5.3 Adding and Subtracting Rational Expressions

Objectives:
- Add and subtract rational expressions.
- Simplify complex fractions.

Adding and subtracting rational expressions is similar to adding and subtracting fractions. To add or subtract rational expressions with like denominators, add or subtract the numerators and use the same denominator.

$$\frac{1}{5} + \frac{3}{5} = \frac{4}{5} \qquad \frac{6}{7} - \frac{4}{7} = \frac{2}{7}$$

Add or subtract.

Ex. 1: $\dfrac{x-3}{x+4} + \dfrac{x-2}{x+4}$

$= \dfrac{2x-5}{x+4}$

Ex. 2: $\dfrac{3x^2-5}{3x-1} - \dfrac{2x^2-3x-2}{3x-1}$

$= \dfrac{3x^2-5-2x^2+3x+2}{3x-1}$

$= \dfrac{x^2+3x-3}{3x-1}$

To add or subtract rational expressions with unlike denominators, first find the least common denominator (LCD). The LCD is the least common multiple (LCM) of the polynomials in the denominators.

Least Common Multiple (LCM) of Polynomials
To find the LCM of polynomials:
1. Factor each polynomial completely. Write any repeated factors as powers. For example, $x^3 + 6x^2 + 9x = x(x+3)^2$.
2. List the different factors. If the polynomials have common factors, use the highest power of each common factor.

NOTE: You can always find a common denominator of two rational expressions by multiplying the denominators. However, when you use the least common denominator, simplifying your answer may take fewer steps.

Find the least common multiple for each pair.

Ex. 3: $4x^2y^3$ and $6x^4y^5$ $\quad 12x^4y^5$

$2 \quad 2$ $\quad 2^1 \cdot 3^1$ $\quad 2^2 \cdot 3^1$

2^2

Take highest power of each factor.

Ex. 4: $x^2 - 2x - 3$ and $x^2 - x - 6$ $\quad$ Factor 1st

$(x+1)^1(x-3)^1 \quad (x-3)^1(x+2)^1$

LCD or LCM

$(x+1)^1(x-3)^1(x+2)^1$

To add rational expressions with unlike denominators, rewrite both expressions with the LCD. This process is similar to adding fractions.

$$\frac{2}{6}+\frac{3}{10}=\frac{2}{2\cdot 3}\left(\frac{5}{5}\right)+\frac{3}{2\cdot 5}\left(\frac{3}{3}\right)$$

$2^1\cdot 3^1$ $\quad$ $3^1\cdot 5^1$

$$=\frac{10}{30}+\frac{9}{30}=\frac{19}{30}$$

Add or Subtract.

Ex. 5: $\dfrac{x-3}{x^2+3x-4}+\dfrac{2x}{x+4}\cdot\dfrac{(x-1)}{(x-1)}$

$(x+4)(x-1)$

$$\frac{x-3}{(x+4)(x-1)}+\frac{2x^2-2x}{(x+4)(x-1)}=\frac{2x^2-x-3}{(x+4)(x-1)}$$

$(2x-3)(x+1)$

Ex. 6: $3\cdot\dfrac{3x}{2x-2}+\dfrac{(3x-2)}{3x-3}\,2$

$3\cdot 2(x-1)$ $\quad$ $3(x-1)\cdot 2$

$$=\frac{9x}{2\cdot 3(x-1)}+\frac{6x-4}{2\cdot 3(x-1)}$$

$$=\frac{15x-4}{2\cdot 3(x-1)}$$

Ex. 7: $\dfrac{2x^2-30}{x^2-9} - \dfrac{x+5}{x+3}$

$(x+3)(x-3) \qquad (x-3)$

$$= \frac{2x^2-30}{(x+3)(x-3)} - \frac{x^2+2x-15}{(x+3)(x-3)}$$

$$+ \; -x^2-2x+15$$

$$= \frac{x^2-2x-15}{(x+3)(x-3)}$$

$$= \frac{(x-5)\cancel{(x+3)}}{\cancel{(x+3)}(x-3)} = \frac{x-5}{x-3}$$

Ex. 8: $\dfrac{x+2}{2x-2} - \dfrac{-2x-1}{x^2-4x+3}$

$2(x-1) \qquad (x-1)(x-3)$

$$\frac{x^2-x-6}{2(x-1)(x-3)} \mp \frac{\pm 2x \pm 1}{2(x-1)(x-3)} = \frac{x^2+x-5}{2(x-1)(x-3)}$$

Some rational expressions are *complex fractions*.
A <u>**complex fraction**</u> contains one or more fractions in its numerator, its denominator, or both. Examples of complex fractions are shown below.

$\dfrac{x+2}{\frac{3}{x}}$	$\dfrac{1+\frac{1}{x}}{4x+5}$	$\dfrac{\frac{x+3}{x}}{\frac{x+4}{7x}}$

Recall that the bar in a fraction represents division. Therefore, you can <u>rewrite</u> a complex fraction <u>as a division problem</u> and then simplify.

Simplify. Assume that all expressions are defined.

Ex. 9: $\dfrac{\dfrac{x+1}{x^2-1}}{\dfrac{x}{x-1}} =$

$$\frac{x+1}{x^2-1} \div \frac{x}{x-1} =$$

$$\frac{\cancel{x+1}}{\underbrace{x^2-1}_{\cancel{(x+1)}\cancel{(x-1)}}} \cdot \frac{\cancel{x-1}}{x} = \frac{1}{x}$$

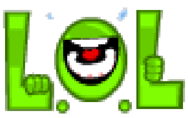
Ex. 10: $\dfrac{\dfrac{1}{x}+\dfrac{1}{2x}}{\dfrac{x+4}{x-2}} =$

$$\left(\frac{1\cdot 2}{x\cdot 2} + \frac{1}{2x}\right) \div \frac{x+4}{x-2} =$$

$$\left(\frac{2}{2x} + \frac{1}{2x}\right) \cdot \frac{x-2}{x+4} =$$

$$\frac{3}{2x} \cdot \frac{x-2}{x+4} = \frac{3(x-2)}{2x(x+4)}$$

Assignment: Practice Worksheets B and C



Teacher: Why didn't you simplify the fraction $\dfrac{\dfrac{x}{x+2}}{\dfrac{x^2}{x-3}}$?

Student: It was too complex.