Algebra 2

8.6 Radical Expressions and Rational Exponents

Objectives:

- Rewrite radical expressions by using rational exponents.
- Simplify and evaluate radical expressions and expressions containing rational exponents.

The *n*th root of a real number *a* can be written as the radical expression $\sqrt[n]{a}$, where *n* is the **index** (plural: *indices*) of the radical and *a* is the **radicand**. When a number has more than one root, the radical sign indicates only the principal, or positive, root.

Reading Math

When a radical sign shows no index, it represents a square root.

All properties of square roots apply to *n*th roots!

To simplify radicals:

Use a factor tree!!

- For **square roots** - pull out **pairs**.
- For **cube roots** - pull out **sets of 3**; **fourth roots** - pull out **sets of 4**, and so on...
- For **variables** - **divide the power by the index**.
- Whatever you have left over stays under the radical.

Ex. 1: Simplify each expression. Assume that all variables are positive.

a. $\sqrt[3]{64x^{12}} = 2\cdot 2\cdot x^4$

$= 4x^4$

$12 \div 3 = 4$

64, 8, 8, 2, 4, 2, 4, 2, 2, 2, 2

b. $\sqrt[4]{64x^{12}} = 2x^3\sqrt[4]{4}$

$12 \div 4 = 3$

2·2·2·2·2·2

c. $\sqrt[5]{64x^{12}}$

$= 2x^2\sqrt[5]{2x^2}$

2·2·2·2·2·2

$5\overline{)12}$ 2 R 2, 10, 2

d. $\sqrt[3]{4x^7}\cdot\sqrt[3]{-2x^2}$

$= \sqrt[3]{-8x^9}$

$= -2x^3$

Remember!

When an expression contains a radical in the denominator, you must rationalize the denominator. To do so, rewrite the expression so that the denominator contains no radicals.

- When rationalizing **square roots** we multiplied by radical by itself **once** (to get a perfect square).
- When rationalizing **cube roots** - multiply by itself **twice** (to get a perfect cube).
- When rationalizing **fourth roots** - multiply by itself **3** times, and so on...

Ex. 2: Simplify.

a. $\sqrt[3]{\frac{x^3}{4}} = \frac{\sqrt[3]{x^3}}{\sqrt[3]{4}} = \frac{x}{\sqrt[3]{4}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}} = \frac{x\sqrt[3]{2}}{\sqrt[3]{8}} = \frac{x\sqrt[3]{2}}{2}$

b. $\sqrt[3]{\frac{27x^9}{y}} = \frac{\sqrt[3]{27x^9}}{\sqrt[3]{y}} = \frac{3x^3}{\sqrt[3]{y}} \cdot \frac{\sqrt[3]{y^2}}{\sqrt[3]{y^2}} = \frac{3x^3\sqrt[3]{y^2}}{y}$

A rational exponent is an exponent that can be expressed as a fraction, $\frac{m}{n}$, where m and n are integers and $n \neq 0$.

Radical expressions can be written by using rational exponents:

$$a^{\frac{1}{n}} = \sqrt[n]{a} \qquad a^{\frac{m}{n}} = \sqrt[n]{a^m} \text{ OR } \left(\sqrt[n]{a}\right)^m$$

Ex. $a^{\frac{1}{3}} = \sqrt[3]{a}$ $\qquad a^{\frac{2}{3}} = \sqrt[3]{a^2} = \left(\sqrt[3]{a}\right)^2$

Writing Math

The denominator of a rational exponent becomes the index of the radical.

Flower Power

$\frac{\text{power}}{\text{root}}$

Ex. 3: Write each expression in **radical form and simplify.**

a. $(-32)^{\frac{3}{5}} = \left(\sqrt[5]{-32}\right)^3$

$= (-2)^3 = -8$

b. $4^{\frac{5}{2}} = \left(\sqrt{4}\right)^5 = 2^5$

$= 32$

Ex. 4: Write each expression by using **rational exponents.**

a. $\sqrt[8]{13^4} = 13^{\frac{4}{8}}$

$= 13^{\frac{1}{2}}$

b. $\sqrt{5} = 5^{\frac{1}{2}}$

Rational Exponents have the same properties as integer exponents:

- **Product of Powers** $a^m \cdot a^n = a^{m+n}$
- **Quotient of Powers** $\dfrac{a^m}{a^n} = a^{m-n}$
- **Power of a Power** $(a^m)^n = a^{m \cdot n}$
- **Power of a Quotient** $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$
- **Power of a Product** $(ab)^m = a^m b^m$
- **Negative Exponents** $a^{-n} = \dfrac{1}{a^n}$ and $\dfrac{1}{a^{-n}} = a^n$

Remember:
When adding/subtracting fractions you need a **common denominator!!**

Ex. 5: Simplify each numerical expression. Leave variable expressions in simplified rational exponent form. Change radicals to rational exponents to make them easier to work with.

a. $\left(7^{\frac{7}{9}}\right)\left(7^{\frac{11}{9}}\right)$

$= 7^{\frac{18}{9}} = 7^2 = 49$

b. $\dfrac{16^{\frac{3}{2}}}{16^{\frac{5}{4}}} = \dfrac{16^{\frac{6}{4}}}{16^{\frac{5}{4}}}$

$= 16^{\frac{1}{4}} = \sqrt[4]{16} = 2$

c. $\left(64^{\frac{1}{2}}\right)^{-\frac{1}{3}} = 64^{-\frac{1}{6}}$

$= \dfrac{1}{\sqrt[6]{64}} = \dfrac{1}{2}$

d. $\left(\dfrac{8^{\frac{1}{2}}}{9^{\frac{3}{4}}}\right)^{\frac{2}{3}} = \dfrac{8^{\frac{1}{3}}}{9^{\frac{1}{2}}}$

$= \dfrac{\sqrt[3]{8}}{\sqrt{9}} = \dfrac{2}{3}$

e. $\left(\dfrac{16^{\frac{1}{2}}}{27^{\frac{2}{3}}}\right)^{-\frac{1}{2}} = \dfrac{16^{-\frac{1}{4}}}{27^{-\frac{1}{3}}}$

$= \dfrac{\sqrt[3]{27}}{\sqrt[4]{16}} = \dfrac{3}{2}$

f. $\left(\sqrt[3]{a}b^{\frac{1}{4}}\right)^{\frac{3}{2}} = \left(a^{\frac{1}{3}}b^{\frac{1}{4}}\right)^{\frac{3}{2}}$

$= a^{\frac{1}{2}}b^{\frac{3}{8}}$