Algebra 2

6.2 Composite Functions

Objectives:
- Evaluate and write composite functions.
- Use composite functions to determine if two functions are inverses.

The **composition of functions** uses the output from one function as the input for a second function. The order of function operations is the same as the order of operations for numbers and expressions.

The composition of functions f and g is notated: $(f \circ g)(x) = f(g(x))$

Reading Math

The composition $(f \circ g)(x)$ or $f(g(x))$ is read: "f of g of x."

To find f(g(3)), evaluate g(3) first and then substitute the result into f.

$3 \xrightarrow{g} g(3) \xrightarrow{f} f(g(3))$

washing machine clothes dryer

Evaluating a Composition of Functions

Ex. 1: Find the indicated value.

Given $f(x) = 2x + 3$ and $g(x) = x^2 + 1$, find each value.

a. $f(g(4))$

$g(4) = 4^2 + 1$
$= 17$
$f(17) = 2(17) + 3$
$= 34 + 3$
$= 37$

b. $g(f(4))$

$f(4) = 2 \cdot 4 + 3$
$= 8 + 3$
$= 11$
$g(11) = 11^2 + 1$
$= 122$

Note: In general, $f(g(x)) \neq g(f(x))$.

c. $(f \circ g)(-3)$

$g(-3) = (-3)^2 + 1$
$= 9 + 1$
$= 10$
$f(10) = 2(10) + 3$
$= 23$

d. $(g \circ f)(-3)$

$f(-3) = 2(-3) + 3$
$= -6 + 3$
$= -3$
$g(-3) = (-3)^2 + 1$
$= 9 + 1 = 10$

Finding Compositions of Functions

You can us algebraic expressions as well as numbers as inputs into functions. To find a rule for $f(g(x))$, substitute the rule for g into f.

The domain of $f(g(x))$ is the set of all x-values such that x is in the domain of g and $g(x)$ is in the domain of f.

Ex. 2: Given $f(x) = x^2 - 4$, $g(x) = 3x - 2$ and $h(x) = \sqrt{x+1}$, write each composite function. State the domain. $x+1 \geq 0$ D: $x \geq -1$

a. $(f \circ g)(x)$

$g(x) = 3x-2$

$f(3x-2) = (3x-2)^2 - 4$

$= (3x-2)(3x-2) - 4$

$= 9x^2 - 12x + 4 - 4$

$= 9x^2 - 12x$

D: all real #s

b. $f(h(x))$ $h(x) = \sqrt{x+1}$

$f(\sqrt{x+1}) = (\sqrt{x+1})^2 - 4$

$= x+1-4$

$= x-3$

c. $g(f(x))$

$f(x) = x^2 - 4$

$g(x^2-4) = 3(x^2-4) - 2$

$= 3x^2 - 12 - 2$

$= 3x^2 - 14$

D: all reals

d. $(h \circ g)(x)$ $g(x) = 3x-2$

$h(3x-2) = \sqrt{3x-2+1}$

$= \sqrt{3x-1}$

Domain: $3x-1 \geq 0$

$3x \geq 1$

$x \geq \frac{1}{3}$

When both a relation and its inverses are functions, the relation is called a **one-to-one function**. In a one-to-one function, each y-value is paired with exactly one x-value.

You can use a composition of functions to verify that two functions are inverses. Because inverse functions "undo" each other, when you compose two inverses the result is the input value x.

Identifying Inverse Functions

WORDS	ALGEBRA	EXAMPLE
If the compositions of two functions equal the input value, the functions are inverses.	If $f(g(x)) = g(f(x)) = x$, then $f(x)$ and $g(x)$ are inverse functions.	$f(x) = 3x$ and $g(x) = \frac{1}{3}x$ $f(g(x)) = 3\left(\frac{1}{3}x\right) = x$ $g(f(x)) = \frac{1}{3}(3x) = x$

Determining Whether Functions Are Inverses

Ex. 3: Determine by composition whether each pair of functions are inverses.

a. $f(x) = 3x - 1$ and $g(x) = \frac{1}{3}x + 1$

$f(g(x)) = 3\left(\frac{1}{3}x + 1\right) - 1$

$= x + 3 - 1$

$= x + 2$ (not x, so these aren't inverses)

b. For $x \neq 1$ or 0, $f(x) = \frac{1}{x-1}$ and $g(x) = \frac{1}{x} + 1$.

$f(g(x)) = \frac{1}{\frac{1}{x} + 1 - 1} = \frac{1}{\left(\frac{1}{x}\right)} = 1 \div \frac{1}{x} = 1 \cdot \frac{x}{1} = x$

$g(f(x)) = \frac{1}{\left(\frac{1}{x-1}\right)} + 1 = 1 \div \frac{1}{x-1} + 1$

$= 1 \cdot \frac{x-1}{1} + 1$

$= x - 1 + 1 = x$

f & g are inverses.