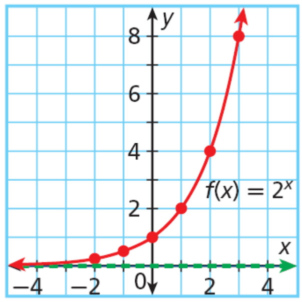
Algebra 2

7.1 Exponential Growth and Decay

Objective: Write and evaluate exponential expressions to model growth and decay situations.

A function of the form $f(x) = ab^x$, with $a > 0$ and $b > 1$, is an **exponential growth** function, which increases as x increases. When $0 < b < 1$, the function is called an **exponential decay** function, which decreases as x increases.

The graph of the parent function $f(x) = 2^x$ is shown. The domain is all real numbers and the range is $y > 0$.



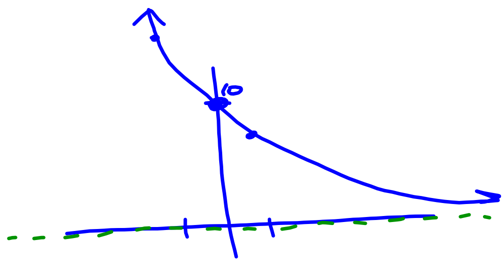
Notice as the x-values decrease, the graph of the function gets closer and closer to the x-axis. The function never reaches the x-axis because the value of 2^x cannot be zero. In this case, the x-axis is an *asymptote*.
An **asymptote** is a line that a graphed function approaches as the value of x gets very large or very small.

Tell whether the function shows growth or decay. Then sketch the graph.

Ex. 1: $f(x)=10\left(\frac{3}{4}\right)^x$

$\frac{3}{4}$ decay

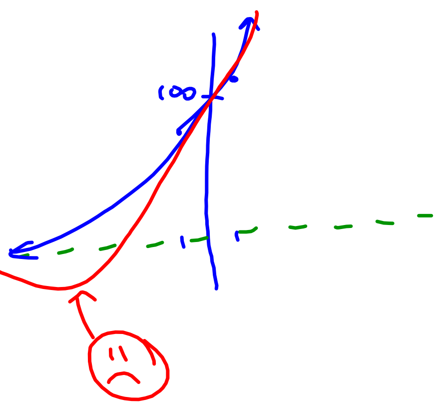
x	y
−1	$13\frac{1}{3}$
0	10
1	7.5



growth

Ex. 2: $g(x) = 100(1.05)^x$

x	y
−1	95.3
0	100
1	105



Ex. 3: A bacteria containing 25 mg doubles each day.

base = 2

a. Write an exponential function.

$25*2^{30}$

2.68435456E10

$$A(t) = 25 \cdot 2^t$$

initial

b. Predict the number of bacteria in 30 days.

$$A(30) = 25 \cdot 2^{30} = 2.68 \times 10^{10}$$

26,800,000,000

c. Approximate how many days would it take for the bacteria to be over 1000 mg.

$$1000 \approx 25 \cdot 2^t$$

t is about 6

You can model growth or decay by a constant percent increase or decrease with the following formula:

Initial amount Number of time periods

$$A(t) = a(1 \pm r)^t$$

Final amount Rate of increase

growth

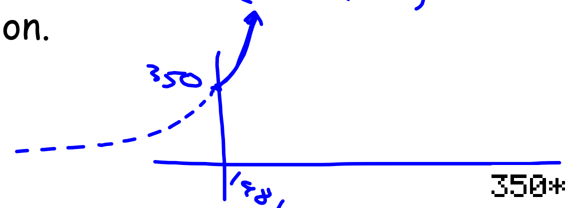
In the formula, the base of the exponential expression, $1 + r$, is called the *growth factor*. Similarly, $1 - r$ is the *decay factor*.

Ex. 4: In 1981, the Australian humpback whale population was 350 and increased at a rate of 14% each year since then.

a. Write a function to model population growth.

$$A(t) = 350(1 + .14)^t$$

b. Graph the function.



c. Find the whale population after 4 years.

```
350*1.14^4
         591.136056
```

$$A(4) = 350(1.14)^4$$

d. Use a graph to predict when the population will reach 20,000.

$$20000 \approx 350(1.14)^t$$

t is roughly 31 yrs

```
          2498.278292
350*1.14^30
          17832.5555
350*1.14^31
          20329.11327
```

Ex. 5:

A motor scooter purchased for $1000 depreciates at an annual rate of 15%. Write an exponential function and graph the function. Use the graph to predict when the value will fall below $100.

$$A(t) = 1000(1 - .15)^t$$

$$= 1000(.85)^t$$

t is about 14 to 15 yrs