

## Solving Nonlinear Systems

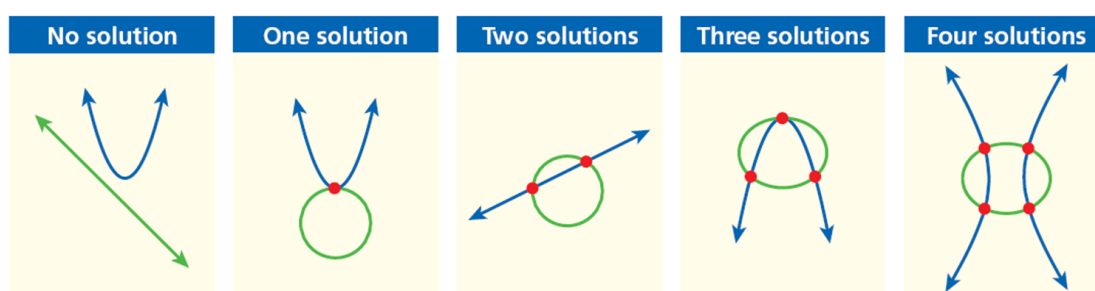
**Objective:** Solve systems of equations in two variables that contain at least one second-degree equation.

**Why...?**

How can law enforcement agencies track a cell phone user's location?

← Using a system of quadratic equations

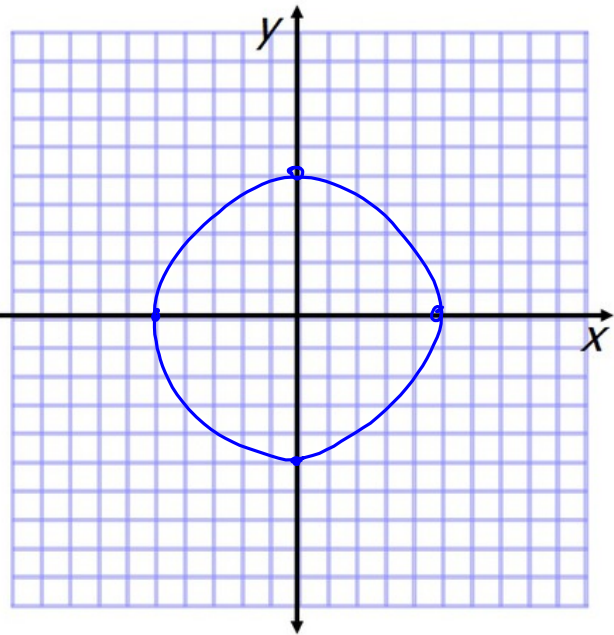
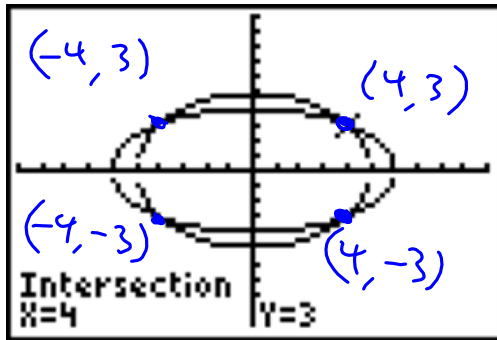
A **nonlinear system of equations** is a system in which at least one of the equations is not linear. You have been studying one class of nonlinear equations, the conic sections.



The solution set of a system of equations is the set of points that make all of the equations in the system true, or where the graphs intersect. For systems of nonlinear equations, you must be aware of the number of possible solutions.

### Example 1: Solving a Nonlinear System by Graphing

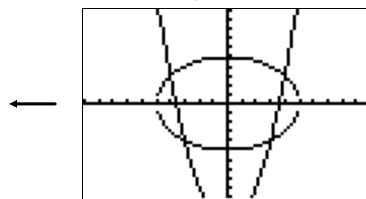
$$\begin{cases} x^2 + y^2 = 25 \\ 4x^2 + 9y^2 = 145 \end{cases}$$



### Example 2: Solving a Nonlinear System by Substitution

$$\begin{aligned} & \begin{cases} x^2 + y^2 = 100 \\ y = \frac{1}{2}x^2 - 26 \end{cases} \cdot 2 \\ & 2y = x^2 - 52 \\ & \quad +52 \quad \quad +52 \\ & \underline{2y + 52 = x^2} \end{aligned}$$

$$\begin{aligned} & x^2 + y^2 = 100 \\ & 2y + 52 + y^2 = 100 \\ & \quad -100 \quad \quad -100 \\ & y^2 + 2y - 48 = 0 \\ & (y+8)(y-6) = 0 \\ & \underline{y = -8} \quad \underline{y = 6} \\ & \begin{aligned} x^2 &= 2(-8) + 52 & x^2 &= 2(6) + 52 \\ x^2 &= -16 + 52 & x^2 &= 12 + 52 \\ \sqrt{x^2} &= \sqrt{36} & x^2 &= 64 \\ \underline{x = \pm 6} & & \underline{x = \pm 8} & \end{aligned} \\ & \begin{aligned} & (6, -8) & & (8, 6) \\ & (-6, -8) & & (-8, 6) \end{aligned} \end{aligned}$$



The graph of the first equation is a circle, and the graph of the second equation is a parabola, so there may be as many as four points of intersection.

**Example 3: Solving a Nonlinear System by Substitution**

$$\begin{cases} x^2 + y^2 = 25 \\ x + y = -1 \end{cases} \quad \begin{aligned} (-1-y)^2 + y^2 &= 25 \\ 1 + 2y + y^2 + y^2 &= 25 \\ -25 & \quad -25 \end{aligned}$$

$$x = -1 - y$$

$$\frac{2y^2 + 2y - 24}{2} = \frac{0}{2}$$

$$y^2 + y - 12 = 0$$

$$(y+4)(y-3) = 0$$

$$y = -4 \quad | \quad y = 3$$

$$x = -1 - (-4) \quad | \quad x = -1 - (3)$$

$$x = -1 + 4 \quad | \quad x = -1 - 3 = -4$$

$$(3, 4) \quad | \quad (-4, 3)$$

**Example 4: Solving a Nonlinear System by Elimination**

$$\begin{cases} 4x^2 + 25y^2 = 41 \\ 36x^2 + 25y^2 = 169 \end{cases} \quad (-1)$$

$$-4x^2 - 25y^2 = -41$$

$$\frac{32x^2}{32} = \frac{128}{32}$$

$$\sqrt{x^2} = \sqrt{4}$$

$$x = \pm 2$$

$$4(4) + 25y^2 = 41$$

$$16 + 25y^2 = 41$$

$$-16 \quad -16$$

$$25y^2 = 25$$

$$y^2 = 1$$

$$y = \pm 1$$

The solution set of the system is  $\{(-2, -1), (-2, 1), (2, -1), (2, 1)\}$ .

**Example 5: Solving a Nonlinear System by Elimination**

$$\begin{cases} 25x^2 + 9y^2 = 225 \\ 25x^2 - 16y^2 = 400 \end{cases} \quad (-1)$$
$$\underline{-25x^2 - 9y^2 = -225}$$

$$-25y^2 = 175$$

$$y^2 = -7$$

no solution

(imaginary #s)